Lecture 4: Welfare Analysis

Mauricio Sarrias

Universidad de Talca
Microeconomía Avanzada I

Version: 2023

Welfare Motivation

- How to measure changes in consumers' welfare?

Welfare Motivation

- How to measure changes in consumers' welfare?
- The first answer is: changes in utility.

Welfare Motivation

- How to measure changes in consumers' welfare?
- The first answer is: changes in utility.

A natural candidate $v(\mathbf{p}^1, y) - v(\mathbf{p}^0, y)$

Welfare Motivation

- How to measure changes in consumers' welfare?
- The first answer is: changes in utility.

A natural candidate $v(\mathbf{p}^1, y) - v(\mathbf{p}^0, y)$

- What is the problem with this?

Welfare Motivation

- How to measure changes in consumers' welfare?
- The first answer is: changes in utility.

A natural candidate $v(\mathbf{p}^1, y) - v(\mathbf{p}^0, y)$

- What is the problem with this?
  - But utility is ordinal: $2u^0$ vs u^0 does not have meaning. The only thing we can say is that if the difference is positive, the consumer likes $(\mathbf{p}^1, y)$ more than $(\mathbf{p}^0, y)$. But, we can't say how much more.

Welfare Motivation

- How to measure changes in consumers' welfare?
- The first answer is: changes in utility.

A natural candidate $v(\mathbf{p}^1, y) - v(\mathbf{p}^0, y)$

- What is the problem with this?
  - But utility is ordinal: $2u^0$ vs u^0 does not have meaning. The only thing we can say is that if the difference is positive, the consumer likes $(\mathbf{p}^1, y)$ more than $(\mathbf{p}^0, y)$. But, we can't say how much more.
  - Furthermore, we cannot add up differences in individuals' utility: 'Interpersonal comparison of utility'

Welfare Motivation

Let me ask you this: By how much would I have to change your wealth so that you are indifferent between $(\mathbf{p}^1, y)$ and $(\mathbf{p}^0, y^1)$?

Welfare Motivation

Let me ask you this: By how much would I have to change your wealth so that you are indifferent between $(\mathbf{p}^1, y)$ and $(\mathbf{p}^0, y^1)$?

That is, for what value y^1 does:

$$v(\mathbf{p}^1, y) = v(\mathbf{p}, y^1)$$

The change in wealth, $y^1 - y$, in essence gives a monetary value for the impact of this change in price.

Welfare Motivation

Let me ask you this: By how much would I have to change your wealth so that you are indifferent between $(\mathbf{p}^1, y)$ and $(\mathbf{p}^0, y^1)$?

That is, for what value y^1 does:

$$v(\mathbf{p}^1, y) = v(\mathbf{p}, y^1)$$

The change in wealth, $y^1 - y$, in essence gives a monetary value for the impact of this change in price.

Solution: We can use the expenditure function to measure welfare changes in $.

Welfare Motivation

Recall that:

1. The UMP leads to $\mathbf{x}(\mathbf{p}, y)$ and $v(\mathbf{p}, y)$, which are at least in principle observable. However, $v(\mathbf{p}, y)$ is not good tool for welfare analysis.
2. The EMP leads to $\mathbf{x}^h(\mathbf{p}, u)$ and $e(\mathbf{p}, u)$, which are based on unobservable u (Also in principle), but provide a good measure for the change in a consumer's welfare following a policy change.
3. The Slutsky equation provides the link between the observable concepts, $\mathbf{x}(\mathbf{p}, y)$ and the useful concepts, $\mathbf{x}^h(\mathbf{p}, y)$

Welfare Motivation

How much money is required to achieve a certain level of utility before and after the price change?

Note that this require fixing the level of utility: But which one: the level of utility achieved by the consumer prior to the change and the level achieved or the level achieved after the change?

Compensating vs Equivalent variation.

Compensating Variation

Definition (Compensating Variation)

Assume that the price of one good increases $\mathbf{p}^0 \to \mathbf{p}^1$ How more money the consumer needs to achieve the same utility at prices $\mathbf{p}^1$ as she had before the price change?

$$u^0 \equiv v(\mathbf{p}^0, y) = v(\mathbf{p}^1, y + CV)$$

What is the change in y (for a consumer facing p_1) to make the utility equal to u^0 (when facing price p_0)?

This can also be written as:

$$\begin{aligned} CV(\mathbf{p}^0, \mathbf{p}^1, y) &= \underbrace{e(\mathbf{p}^1, u^0)}_{\text{min \$ to achieve } u^0 \text{ at price } \mathbf{p}^1} - \underbrace{e(\mathbf{p}^0, u^0)}_{\text{min \$ to achieve } u^0 \text{ at price } \mathbf{p}^0} \\ &= e(\mathbf{p}^1, v(\mathbf{p}^0, y)) - e(\mathbf{p}^0, v(\mathbf{p}^0, y)) \\ &= e(\mathbf{p}^1, u^0) - y \end{aligned}$$

That is, if prices change from $\mathbf{p}^0$ to $\mathbf{p}^1$, the magnitude of compensating variation tell us how much we will have to change or compensate our consumer to have her to stay on the same indifference curve.

If prices increases then $CV > 0$

Equivalent Variation

Definition (Equivalent Variation)

Equivalent variation gives the change in expenditure that would be required at the original prices to have the same ("equivalent") effect on consumer as the price change had.

$$u^1 \equiv v(\mathbf{p}^1, y) = v(\mathbf{p}^1, y - EV)$$

What is the change in y (for a consumer facing p_0) to make the utility equal to u^1 (when facing price p_0)?

This can also be written as:

$$\begin{aligned} EV(\mathbf{p}^0, \mathbf{p}^1, y) &= e(\mathbf{p}^1, u^1) - e(\mathbf{p}^0, u^1) \\ &= e(\mathbf{p}^1, v(\mathbf{p}^1, y)) - e(\mathbf{p}^0, v(\mathbf{p}^1, y)) \\ &= y - e(\mathbf{p}^0, v(\mathbf{p}^1, y)) \end{aligned}$$

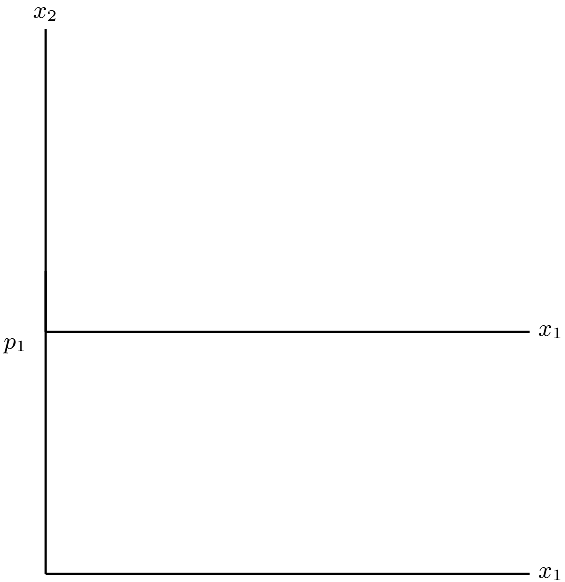
If prices increases then $EV > 0$

Other useful expressions

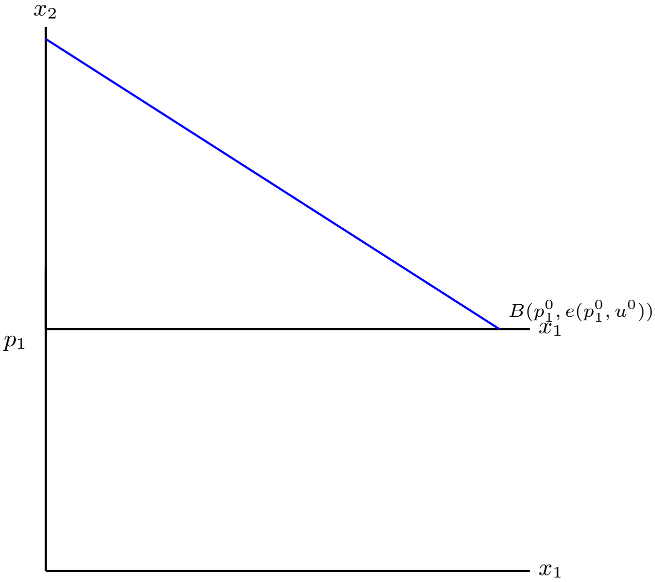
$$CV = e(\mathbf{p}^1, u^0) - e(\mathbf{p}^0, u^0) = \int_{p_i^0}^{p_i^1} \frac{\partial e(\mathbf{p}, u^0)}{\partial p_i} = \int_{p_i^0}^{p_i^1} x_i^h(\mathbf{p}, u^0) dp_i$$

$$EV = e(\mathbf{p}^1, u^1) - e(\mathbf{p}^0, u^1) = \int_{p_i^0}^{p_i^1} \frac{\partial e(\mathbf{p}, u^1)}{\partial p_i} = \int_{p_i^0}^{p_i^1} x_i^h(\mathbf{p}, u^1) dp_i$$

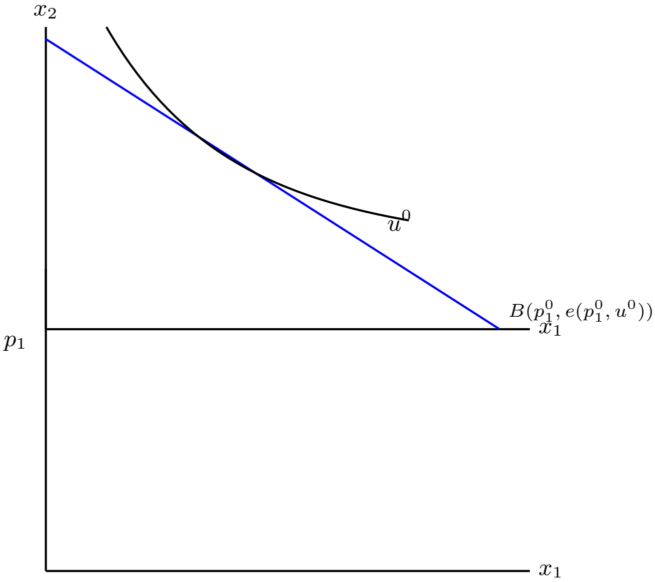
Compensated Variation



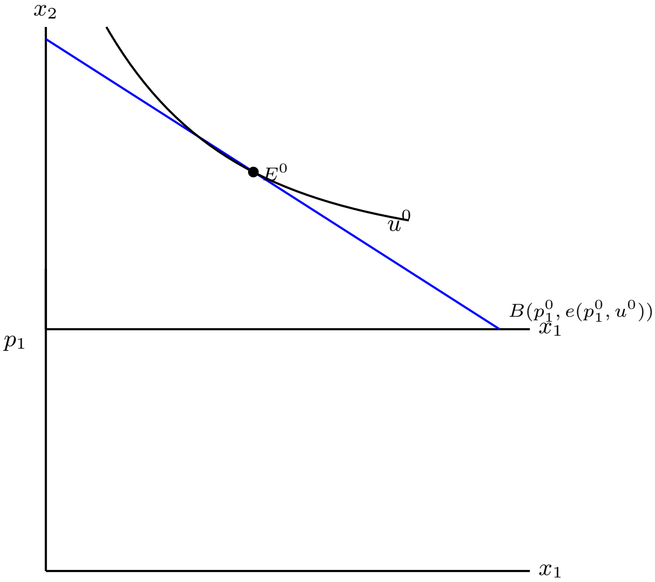
Compensated Variation



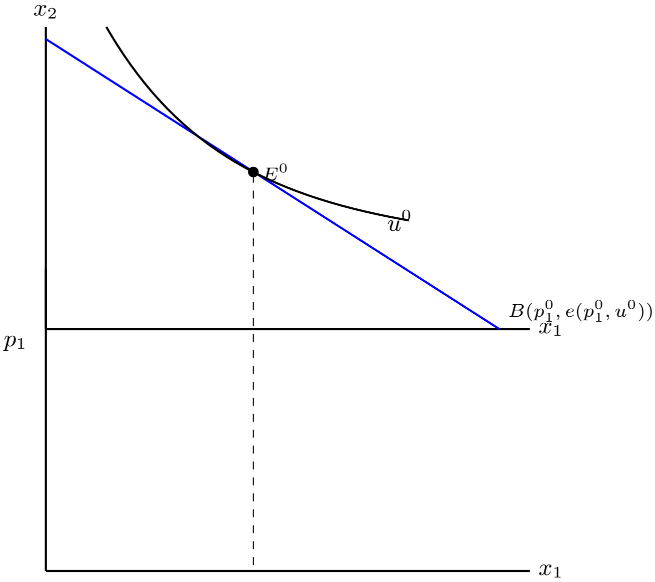
Compensated Variation



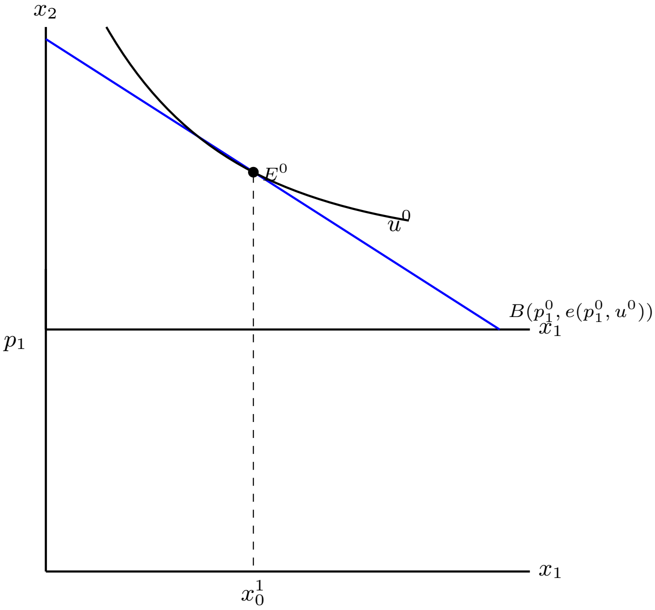
Compensated Variation



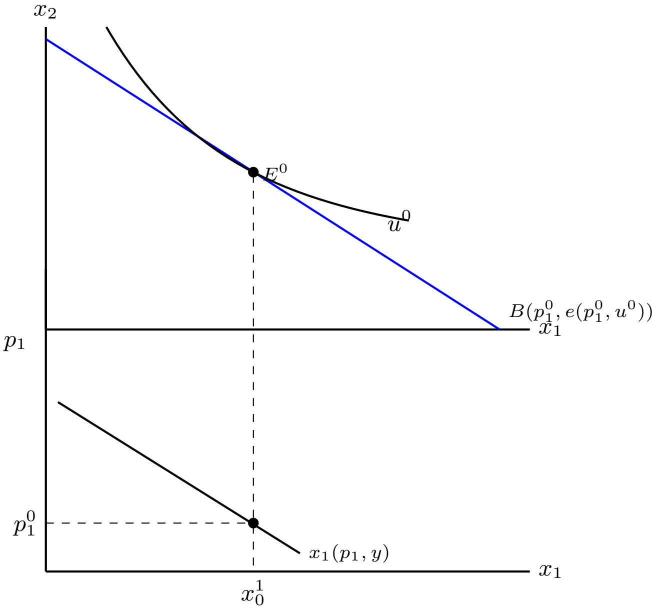
Compensated Variation



Compensated Variation

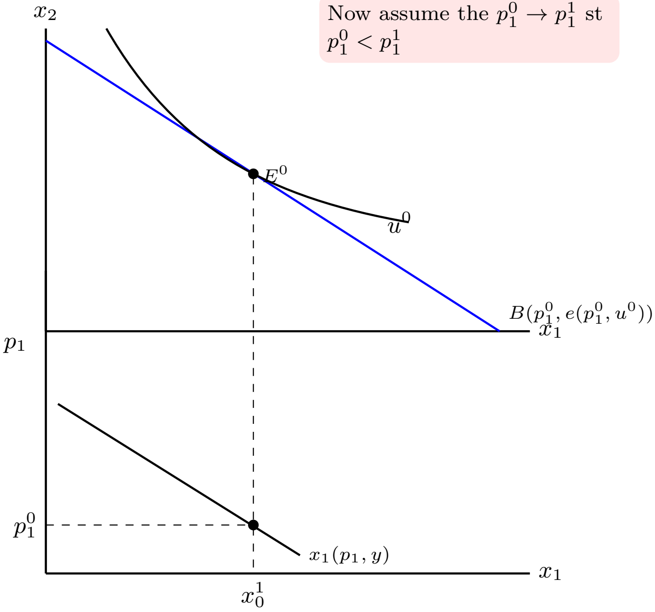


Compensated Variation



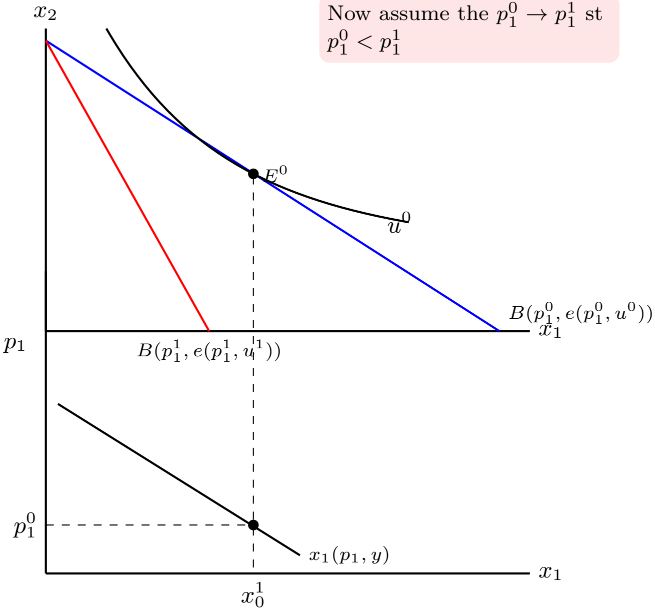
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



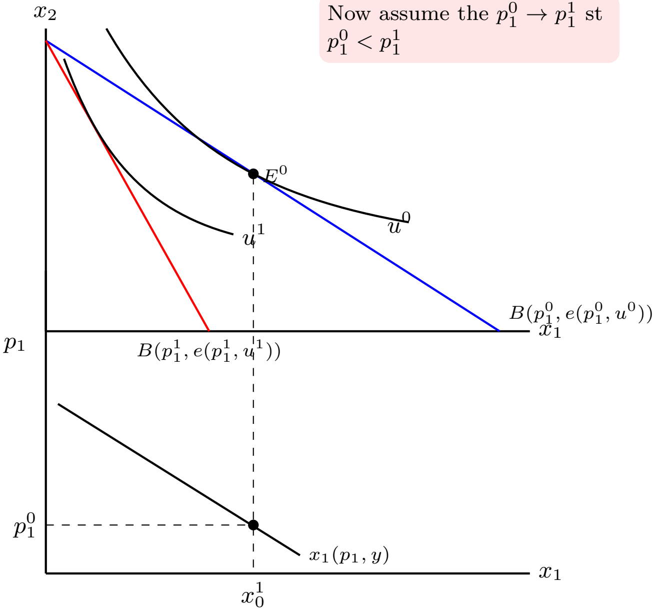
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



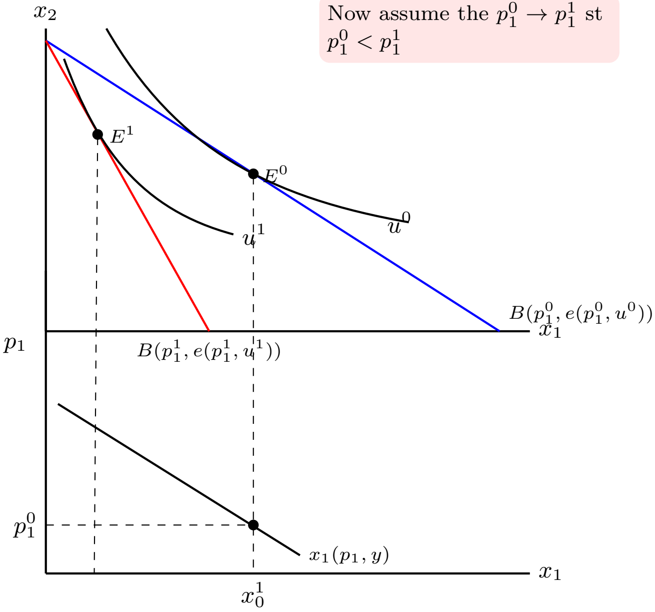
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



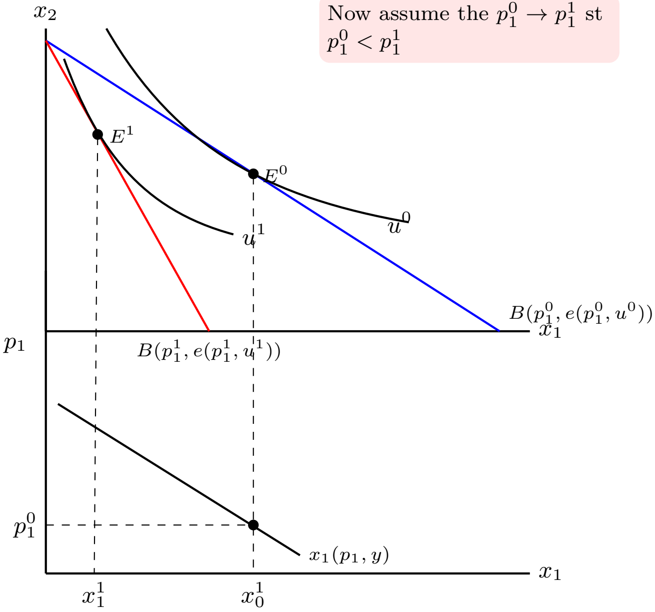
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



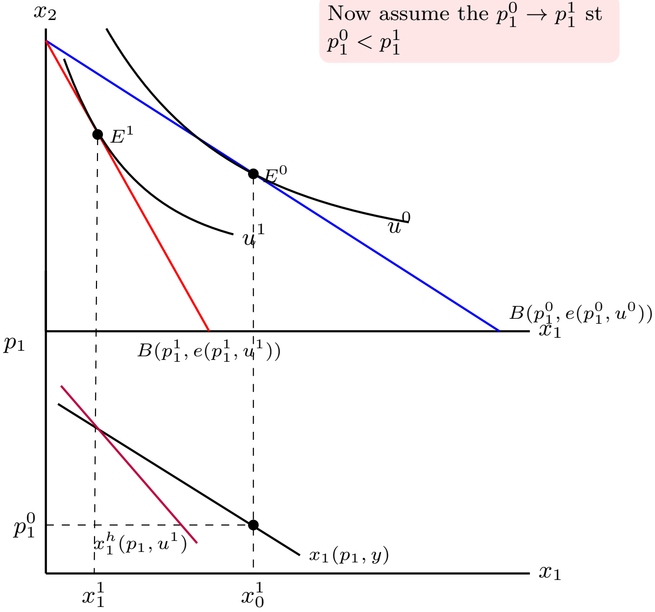
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



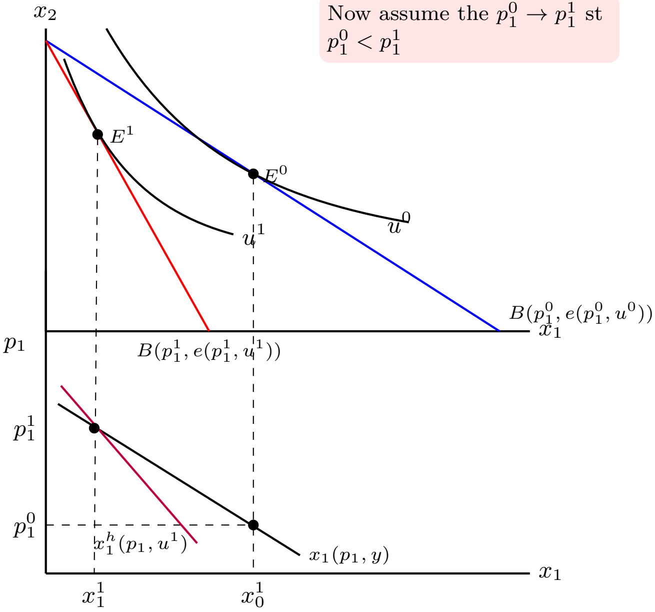
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



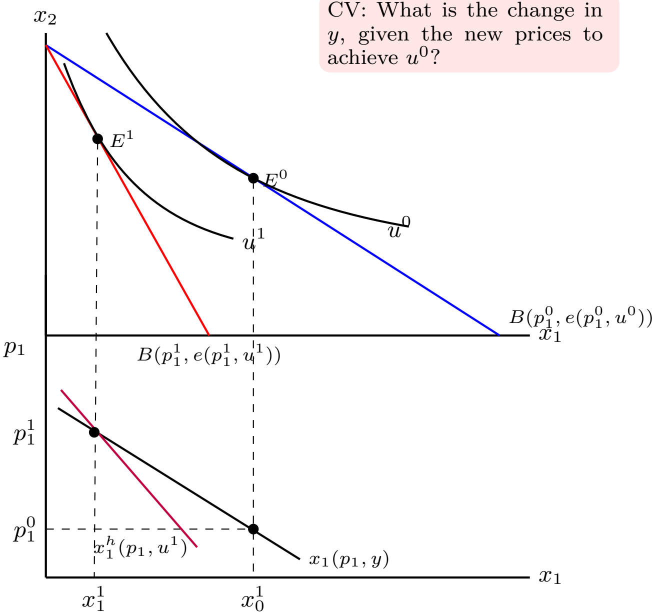
Compensated Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



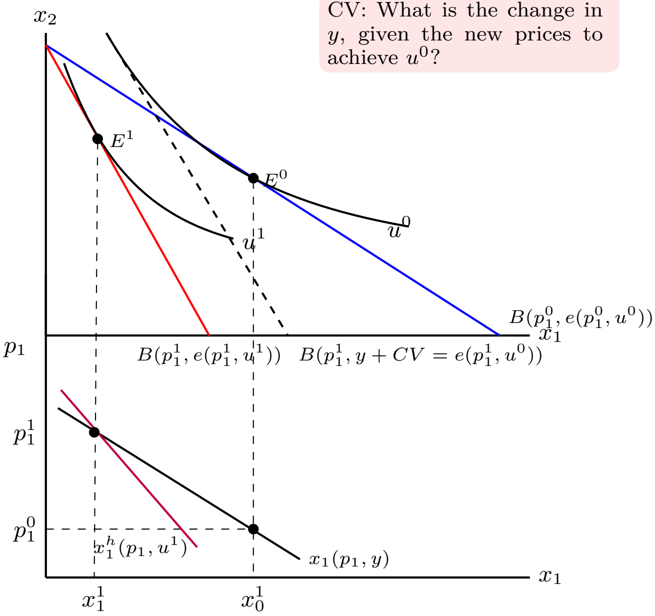
Compensated Variation

CV: What is the change in y, given the new prices to achieve u^0?



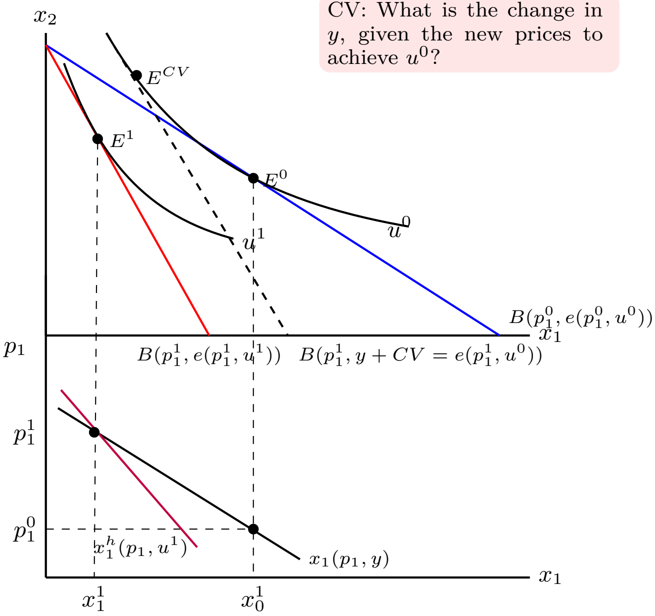
Compensated Variation

CV: What is the change in y, given the new prices to achieve u^0?



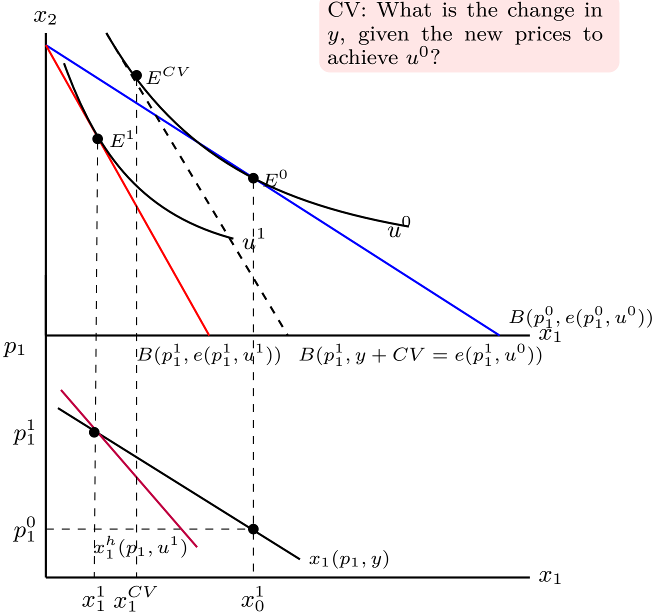
Compensated Variation

CV: What is the change in y, given the new prices to achieve u^0?



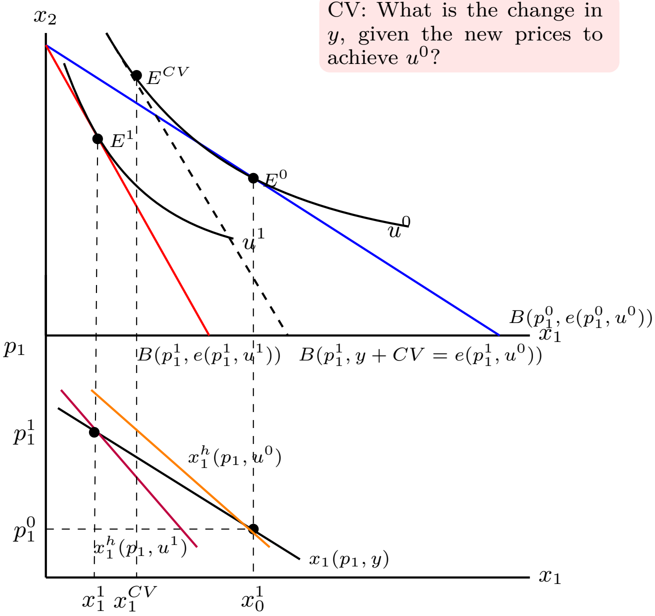
Compensated Variation

CV: What is the change in y, given the new prices to achieve u^0?



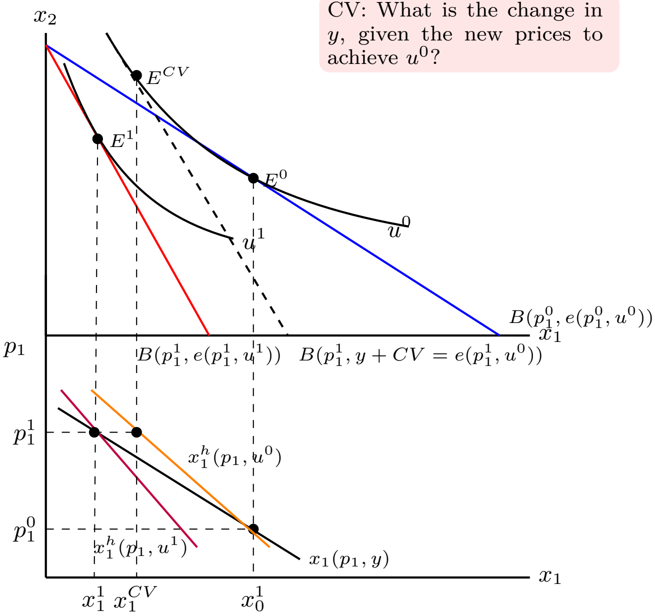
Compensated Variation

CV: What is the change in y, given the new prices to achieve u^0?



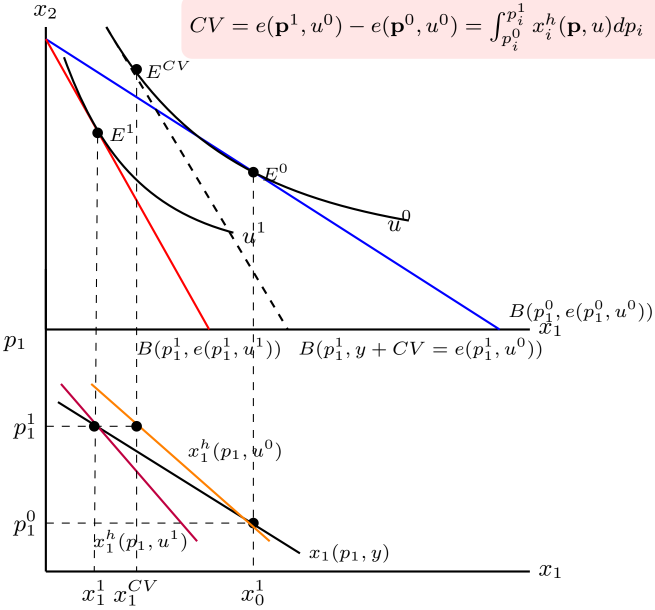
Compensated Variation

CV: What is the change in y, given the new prices to achieve u^0?



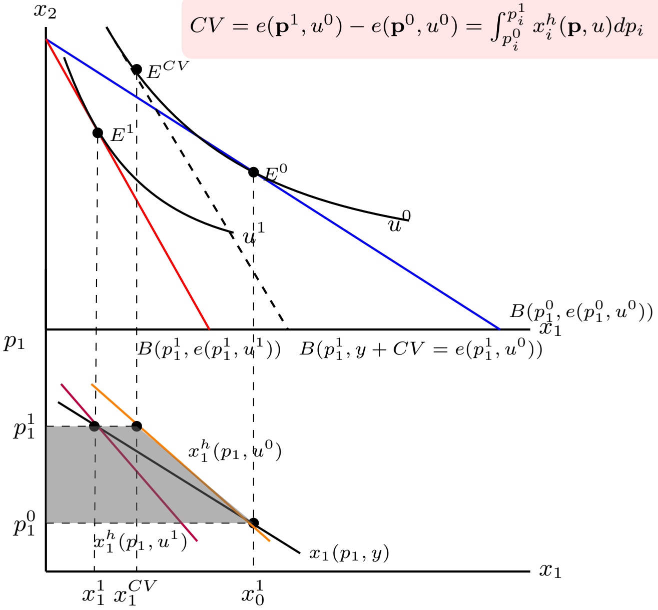
Compensated Variation

$$CV = e(\mathbf{p}^1, u^0) - e(\mathbf{p}^0, u^0) = \int_{p_i^0}^{p_i^1} x_i^h(\mathbf{p}, u) dp_i$$



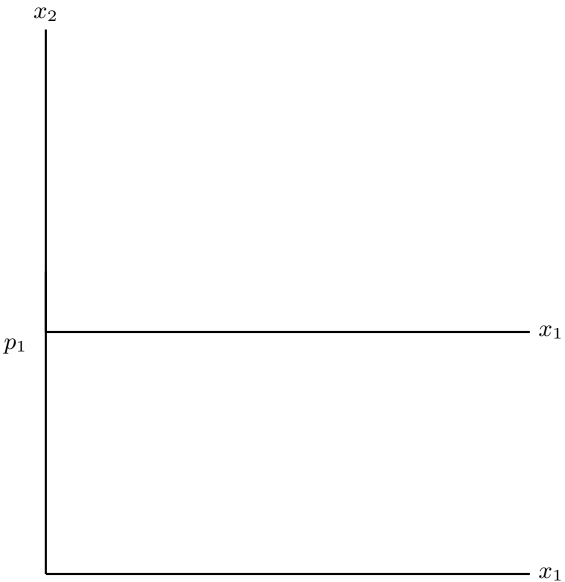
Compensated Variation

$$CV = e(\mathbf{p}^1, u^0) - e(\mathbf{p}^0, u^0) = \int_{p_i^0}^{p_i^1} x_i^h(\mathbf{p}, u)dp_i$$

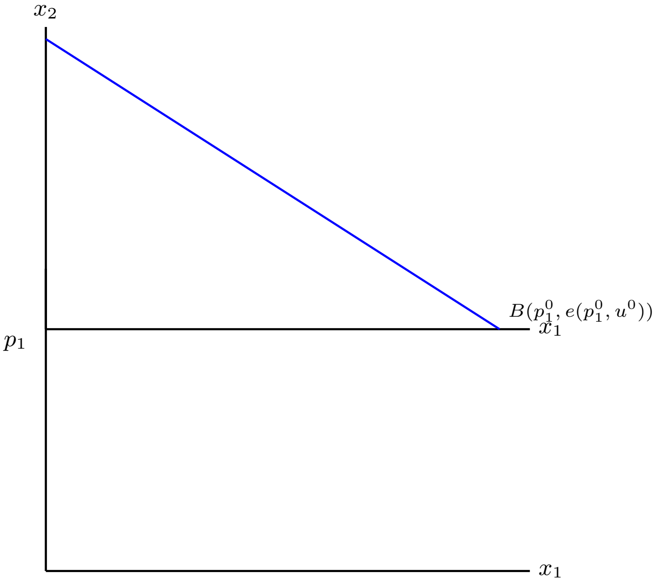


Now Equivalent Variation

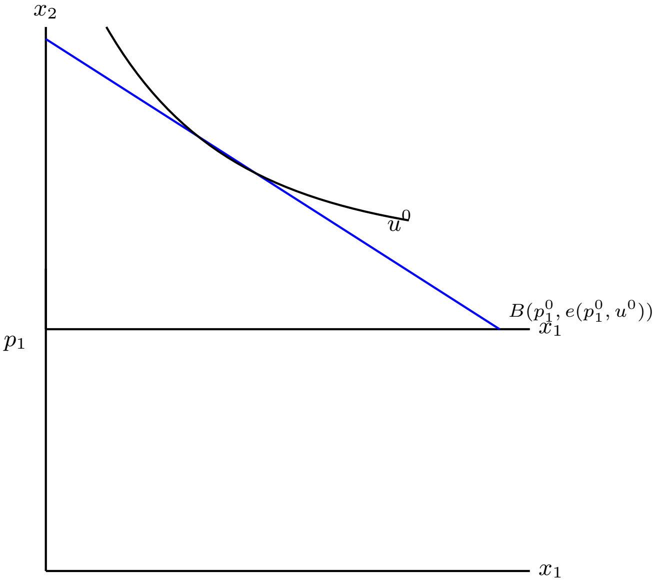
Equivalent Variation



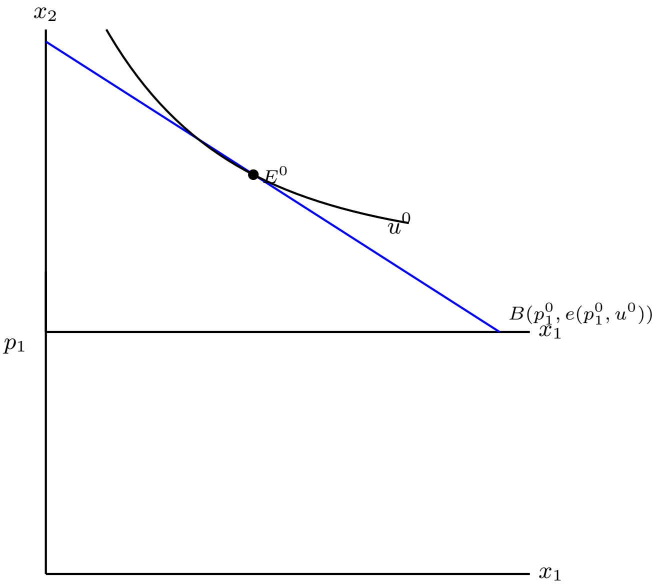
Equivalent Variation



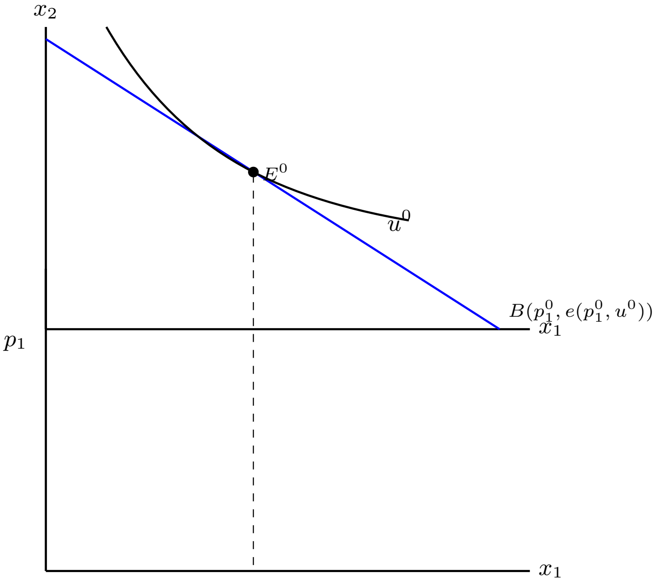
Equivalent Variation



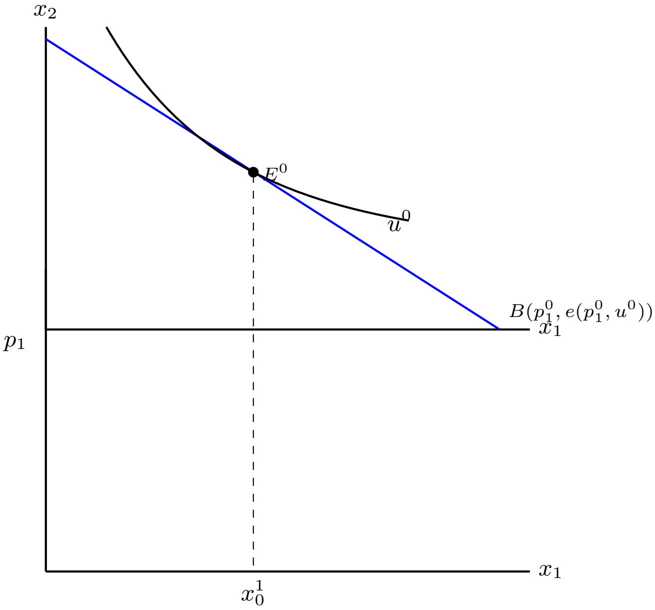
Equivalent Variation



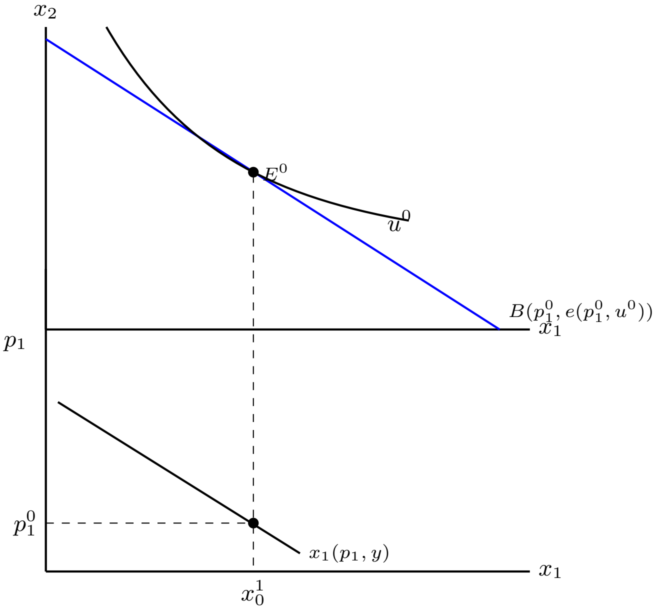
Equivalent Variation



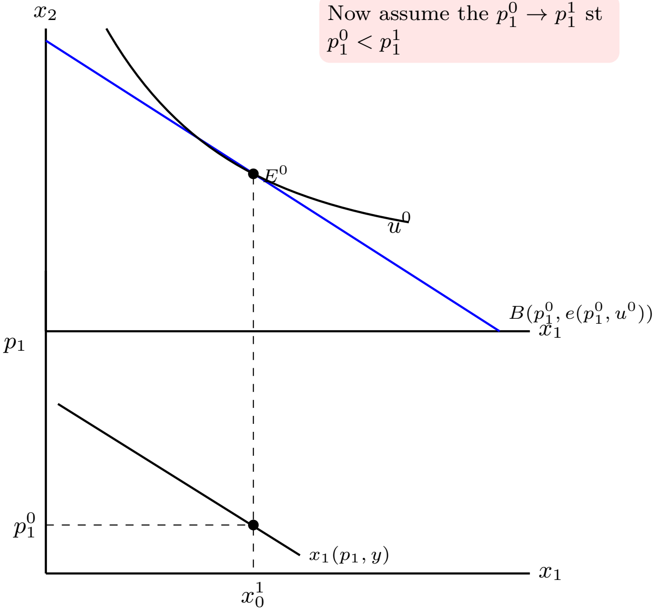
Equivalent Variation



Equivalent Variation



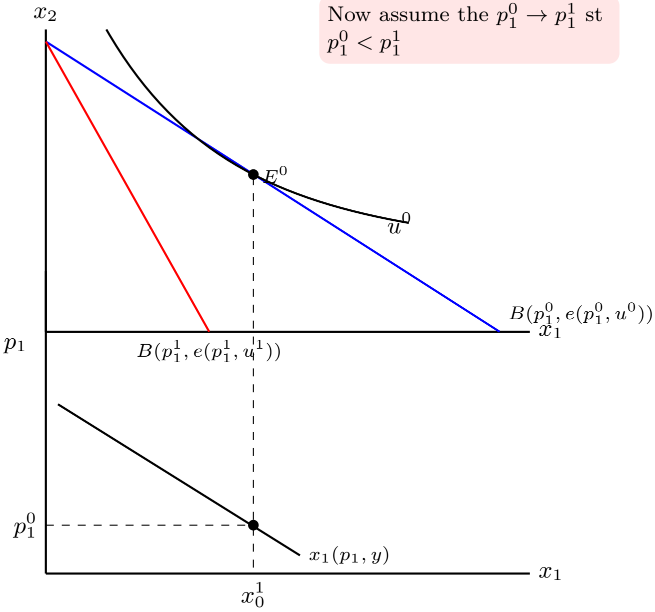
Equivalent Variation



Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$

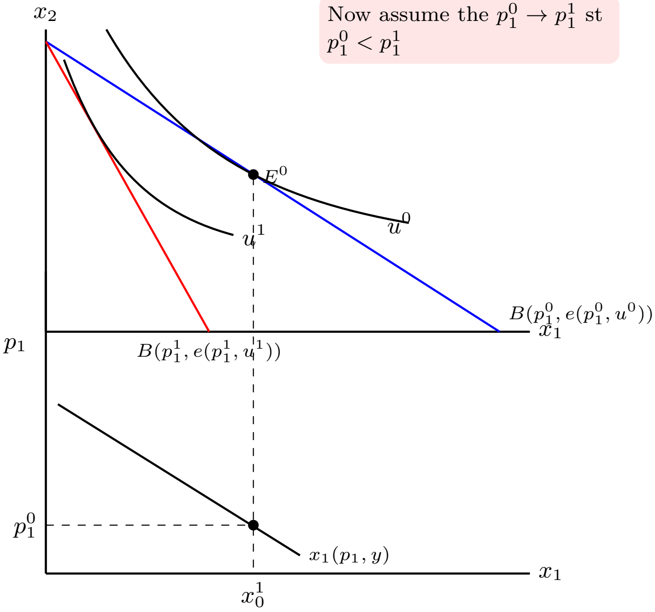
Equivalent Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



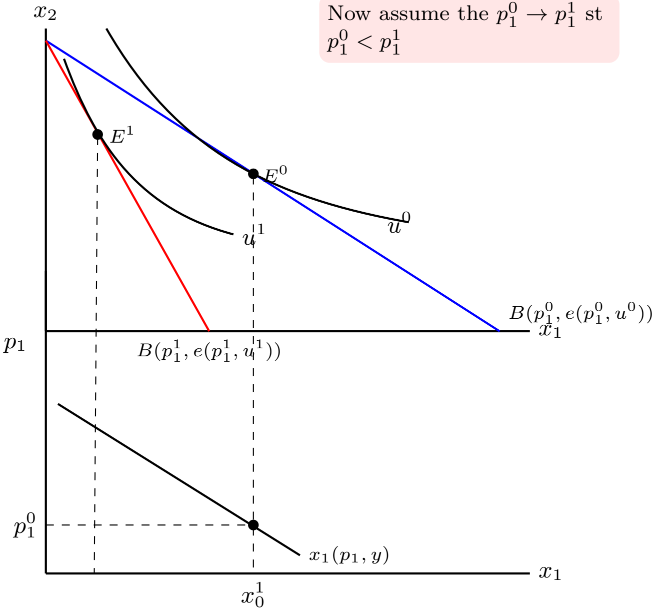
Equivalent Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



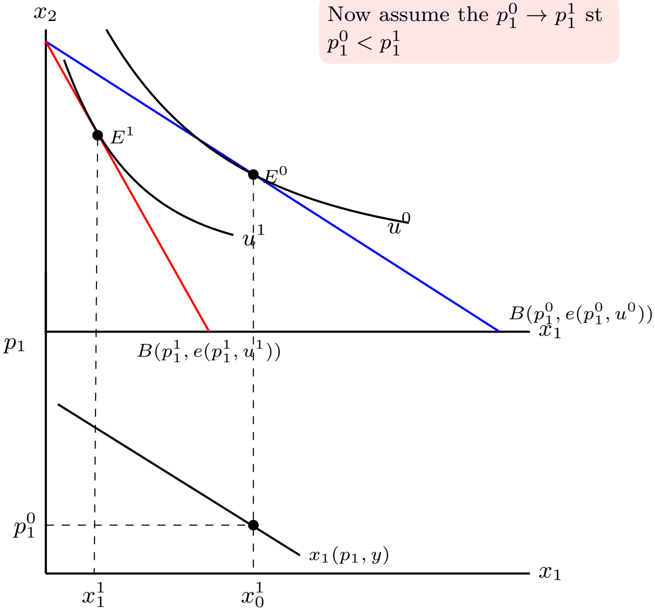
Equivalent Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



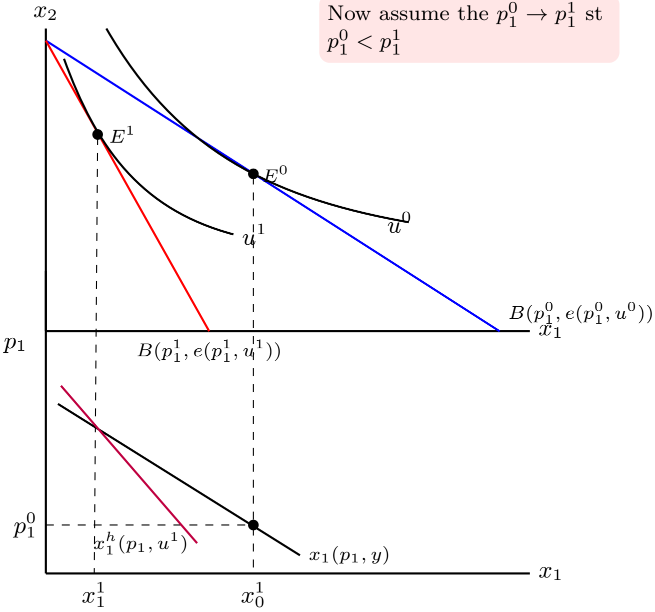
Equivalent Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$

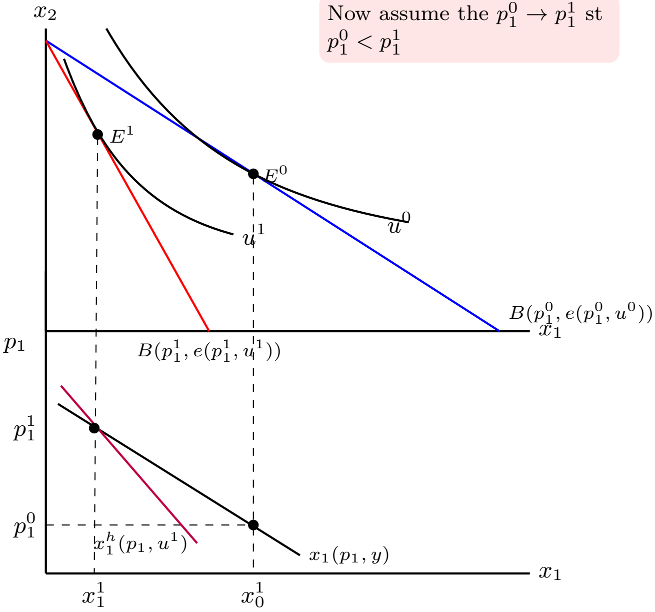


Equivalent Variation

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$



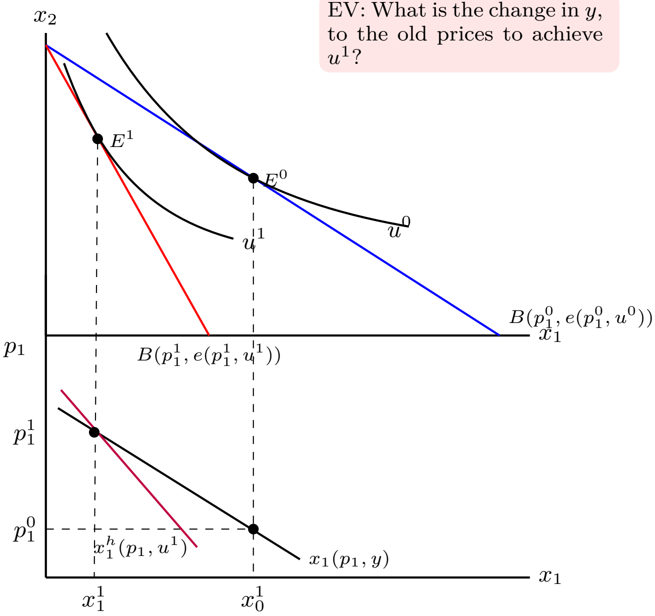
Equivalent Variation



Now assume the $p_1^0 \to p_1^1$ st $p_1^0 < p_1^1$

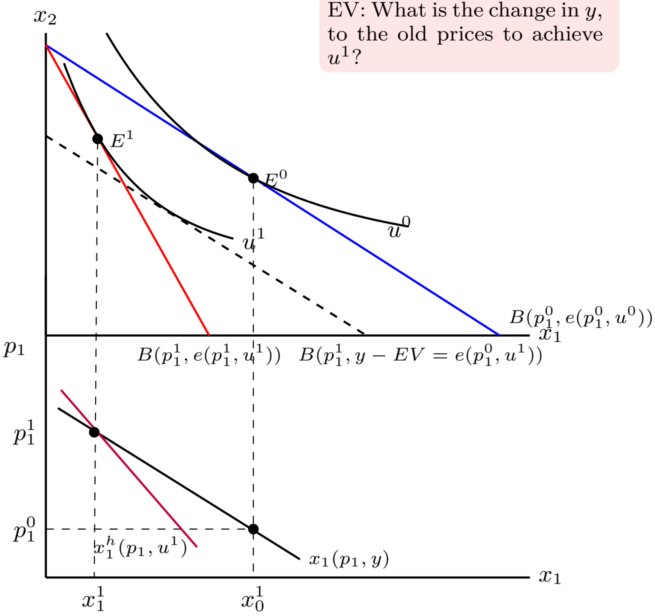
Equivalent Variation

EV: What is the change in y, to the old prices to achieve u^1?



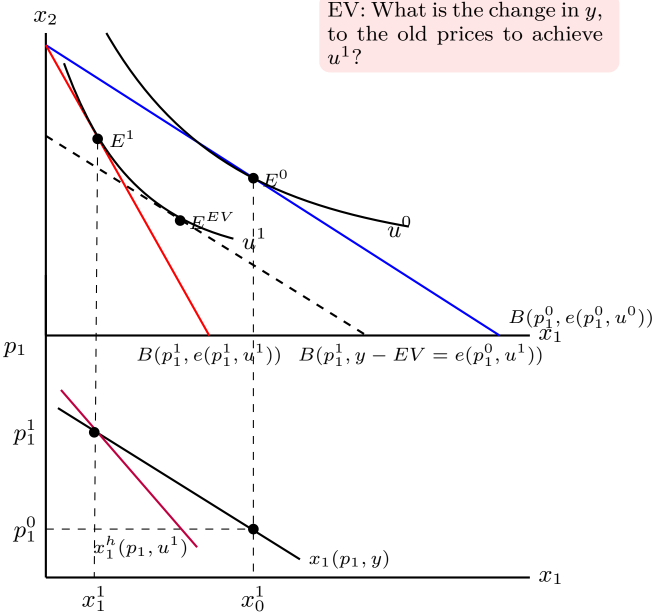
Equivalent Variation

EV: What is the change in y, to the old prices to achieve u^1?



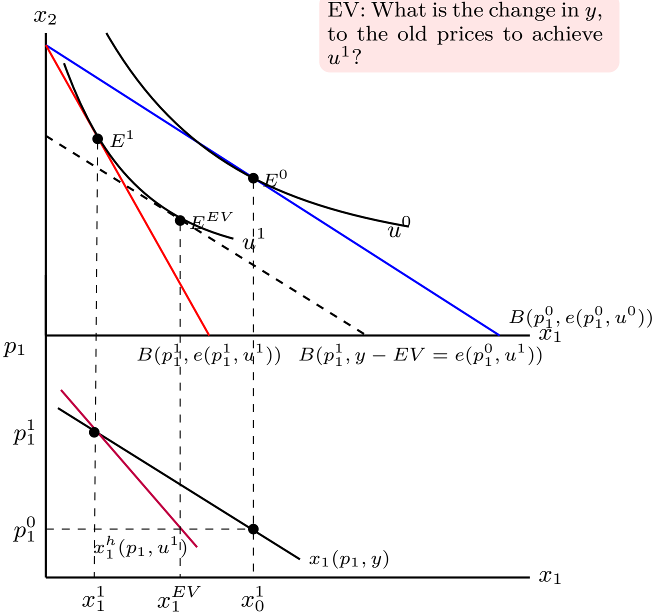
Equivalent Variation

EV: What is the change in y, to the old prices to achieve u^1?



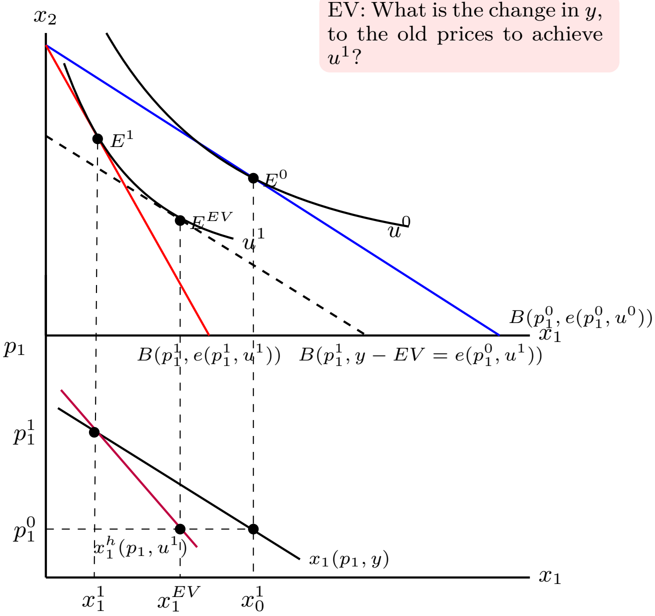
Equivalent Variation

EV: What is the change in y, to the old prices to achieve u^1?



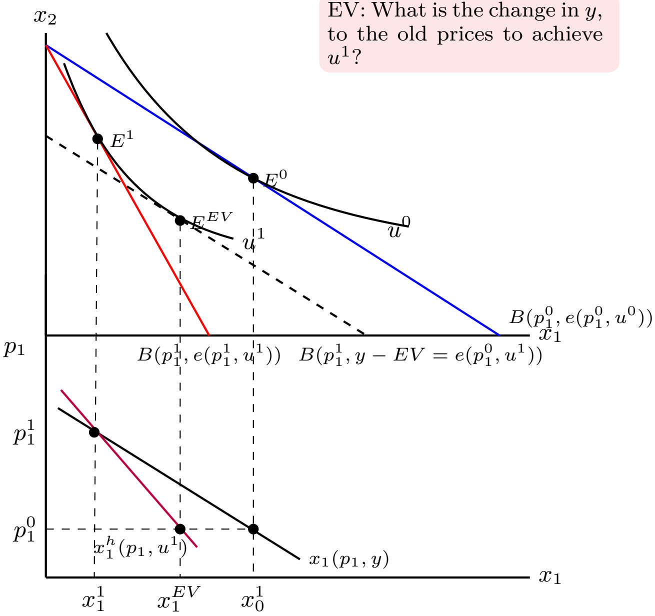
Equivalent Variation

EV: What is the change in y, to the old prices to achieve u^1?



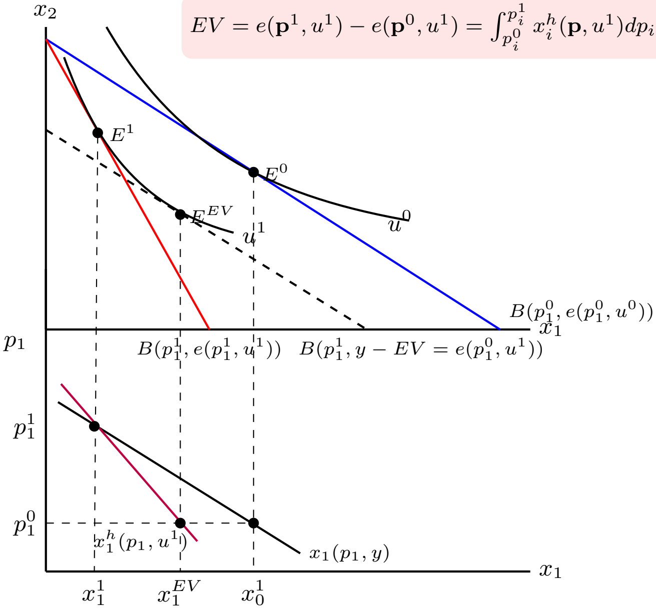
Equivalent Variation

EV: What is the change in y, to the old prices to achieve u^1?



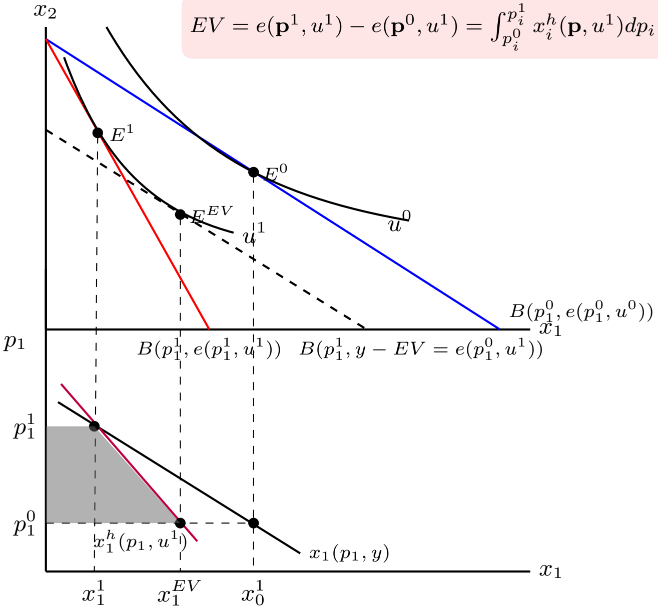
Equivalent Variation

$$EV = e(\mathbf{p}^1, u^1) - e(\mathbf{p}^0, u^1) = \int_{p_i^0}^{p_i^1} x_i^h(\mathbf{p}, u^1) dp_i$$



Equivalent Variation

$$EV = e(\mathbf{p}^1, u^1) - e(\mathbf{p}^0, u^1) = \int_{p_i^0}^{p_i^1} x_i^h(\mathbf{p}, u^1) dp_i$$



Consumer Surplus

- More common way to examine changes in consumer welfare. Why?
- It is the area to the left of the Walrasian demand curve.

$$CS = \int_{p_i^1}^{p_i^0} x_i(p_i, y) dp_i$$

- When good i is a normal good, Walrasian consumer surplus overstate compensating variation and understate equivalent variation for both increases and decreases in p_i: $EV < EC < CV$
- If good i is an inferior good, Walrasian consumer surplus understate compensating variation and overstate equivalent variation for both increases and decreases in p_i: $CV < EC < EV$
- If $\partial x_i / \partial y = 0$: $CV = EC = EV$

Example

Example

Let $u(\mathbf{x}) = x_1^{1/2} x_2^{1/2}$. Assume $p_1^0 = 10, p_2 = 20^0, y = 10$. Suppose that $p_1^1 = 2p_1^0$. Compute EV, CV and CS.

Taxation

- Tax: Government taxes commodity 1, setting a tax on the consumer's purchases of good 1 of t per unit. $p_1^1 = p_1^0 + t$.
- Total Revenue: $T = tx_1(p^1, y)$
- Alternative: A lump-sum tax T is a tax that is a fixed amount in the consumer income, no matter the change in circumstance of the taxed entity.
- Which one is worse?
- The consumer is no worse off and almost always strictly better off under the lump-sum tax (than under an equal revenue yield commodity tax)

Taxation

- Loss in utility due to commodity tax in \$? $\implies$ Equivalent Variation.
- Loss in \$ due to lump-sum tax: T

So, if lump-sum is better, then the loss of commodity tax is greater than loss under lump-sum tax:

$$
\begin{aligned}
EV &> T \\
e(p^1, u^1) - e(p^0, u^1) &> T \\
y - e(p^0, u^1) &> T \\
\underbrace{y - T}_{\text{wealth after LS tax}} &> \underbrace{e(p_0, u^1)}_{\text{commodity tax}}
\end{aligned}
$$

Deadweight loss (DWL) of commodity taxation: $y - T - e(p^0, u^1)$

Taxation

$$\begin{aligned}
e(p^1, u^1) - e(p^0, u^1) - T &= \int_{p^0}^{p^1} x_1^h(p_1, u^1)dp_1 - T \\
&= \int_{p^0}^{p^0+t} x_1^h(p_1, u^1)dp_1 - t \cdot x_1(p^1, y) \\
&= \int_{p^0}^{p^0+t} x_1^h(p_1, u^1)dp_1 - t \cdot x_1(p^1, e(p^1, u^1)) \\
&= \int_{p^0}^{p^0+t} x_1^h(p_1, u^1)dp_1 - t \cdot x_1^h(p^1, u^1) \\
&= \int_{p^0}^{p^0+t} x_1^h(p_1, u^1)dp_1 - t \cdot x_1^h(p^0 + t, u^1) \\
&= \int_{p^0}^{p^0+t} x_1^h(p_1, u^1)dp_1 - \int_{p^0}^{p^0+t} x_1^h(p_0 + t, u^1)dp_1 \\
&= \int_{p^0}^{p^0+t} \left[x_1^h(p_1, u^1) - x_1^h(p_0 + t, u^1)\right] dp_1
\end{aligned}$$

Taxation

Let's see some graphs!

Taxation

Example

The indirect utility function for a consumer is given by: $v(\mathbf{p}, y) = \frac{2y}{2p_1 + p_2}$. The government imposes a tax of t on the first good. It does not returns its tax revenues to the consumer in form of a lump-sum transfer or any in-kind benefit. Find the DWL.

- Thus $DWL = EV - tx_1(p_1^0 + t, p_2^0, y^0)$.
- Use Roy's identity to find the Walrasian Demand: $x_1(p_1^0 + t, p_2^0, y^0) = \frac{2y^0}{2(p_1^0 + t) + p_2^0}$.
- Find the expenditure function using duality: $e(\mathbf{p}, u) = \frac{u(2p_1 + p_2)}{2}$.
- Find $EV = y - e(p^0, u^1) = y - \frac{v(y^0, p^1)(2p_1 + p_2)}{2} = y - \frac{y^0(2p_1^0 + p_2^0)}{2(p_1^0 + t) + p_2^0}$.
- So, $DWL = 0$.

Path independence

- Hicks (1981), what if more than one price is changing?
- Is the welfare measure, whatever is its definition, independent of a particular path that prices change? Example: $(p_1^0, p_2^0) \rightarrow (p_1^1, p_2^0) \rightarrow (p_1^1, p_2^1)$ or $(p_1^0, p_2^0) \rightarrow (p_1^0, p_2^1) \rightarrow (p_1^1, p_2^1)$?
- Graph.
- Path does not matter.

Path independence

Example

Consider the path: $(p_1^0, p_2^0) \to (p_1^1, p_2^0) \to (p_1^1, p_2^1)$
Then:

$$\begin{aligned} CV &= e(\mathbf{p}^0, u^0) - e(\mathbf{p}^1, u^0) \\ &= \left[e(p_1^1, p_2^0, u^0) - e(p_1^0, p_2^0, u^0)\right] + \left[e(p_1^1, p_2^1, u^0) - e(p_1^1, p_2^0, u^0)\right] \\ &= \int_{p_1^0}^{p_1^1} x_1^h(p_1, p_2, u^0) dp_1 + \int_{p_2^0}^{p_2^1} x_1^h(p_1^1, p_2, u^0) dp_2 \end{aligned}$$