Lecture 3: Comparative Statics

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Microeconomía Avanzada I

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1. Comparative Statics
    - Relative Prices and Real Income
    - Some Elasticity Relations
    - Income and Substitution Effects

1 Comparative Statics
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Relative price and real income

- Real vs monetary measures.
  - Relative prices and real income.
- Relative price: number of units of some other good that must be forgone to acquire 1 unit of the good in equation

$$\frac{p_i}{p_j} = \frac{\$/\text{unit } i}{\$/\text{unit } j} = \frac{\$}{\text{unit } i} \cdot \frac{\text{unit } j}{\$} = \frac{\text{unit } j}{\text{unit } i}$$

- Real income: maximum number of units of some commodity the consumer could acquire if the spent his entire money income.

$$\frac{y}{p_j} = \frac{\$}{\$/\text{unit } j} = \text{unit } j$$

- Relative prices and real income affect behaviour $\implies$ absence of money illusion.

Relative price and real income

- Homogeneity allows us to completely eliminate the yardstick of money from any analysis of demand behaviour.
- Trick: arbitrarily designating one of the n goods to serve as numeraire in place of money.
- If its money price is p_n, we can set $t = 1/p_n$ and, invoking homogeneity, conclude that

$$\mathbf{x}(\mathbf{p}, y) = \mathbf{x}(t\mathbf{p}, ty) = \mathbf{x}\left(\frac{p_1}{p_n}, ..., \frac{p_{n-1}}{p_n}, 1, \frac{y}{p_n}\right) \tag{1}$$

In words, demand for each of the n goods depends only on $n-1$ relative prices and the consumer's real income.

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Some Elasticity Relations

Assume that $y = \sum_{i=1}^{n} p_i x_i(\mathbf{p}, y)$ so, all consumer demand responses to price and income changes must add up, or aggregate, in a way that preserves the equality of the budget constraint after the change.

Theorem (Demand Elasticities and Income Shares)

Let $x_i(\mathbf{p}, y)$ be the consumer's Walrasian demand for good i. Then let

$$\eta_i \quad \equiv \frac{\partial x_i(\mathbf{p},y)}{\partial y} \frac{y}{x_i(\mathbf{p},y)}, \epsilon_{ij} \quad \equiv \frac{\partial x_i(\mathbf{p},y)}{\partial p_j} \frac{p_j}{x_i(\mathbf{p},y)}$$

and let

$$s_i \equiv \frac{p_i x_i(\mathbf{p},y)}{y} \quad \text{so that} \quad s_i \geq 0 \quad \text{and} \quad \sum_{i=1}^{n} s_i = 1$$

Some Elasticity Relations

- Income-Elasticity:
  - $\eta_i < 0$: Inferior goods,
  - $\eta_i > 0$: Normal goods,
  - $\eta_i < 1$: Necessity good,
  - $\eta_i > 1$: Luxury good,
  - $\eta_i = 0$: Sticky Goods.
- Price-Elasticity:
  - $\epsilon_{ii} = 0$: Perfect Inelastic,
  - $-1 < \epsilon_{ii} < 0$: Inelastic or relatively inelastic demand,
  - $\epsilon_{ii} = -1$: Unitary elastic,
  - $-\infty < \epsilon_{ii} < -1$: Elastic or relatively elastic demand:
  - $\epsilon_{ii} - \infty$: Perfectly elastic demand
- Cross-Price-Elasticity:
  - $\epsilon_{ij} < 0$: complementary goods:
  - $\epsilon_{ij} > 0$: Substitutes goods.

Some Elasticity Relations

Theorem (Aggregation in Consumer Demand)

Let $\mathbf{x}(\mathbf{p}, y)$ be the consumer's Walrasian demand system. Then the following relation must hold among income shares, price, and income elasticities of demand

1. *Engel aggregation:* $\sum_{i=1}^{n} s_i \eta_i = 1$,
2. *Cournot aggregation:* $\sum_{i=1}^{n} s_i \epsilon_{ij} = -s_j$, j = *1,...,n.*

Engel Aggregation

This condition tells us that at a certain percentage change of monetary income, the average weighted consumption of all goods should vary in the same direction and proportion.

Corollary

If a consumer faces the consumption goods n, they can not all be inferior, at least one must be normal.

Some Elasticity Relations

Corollory Cournot Aggregation

By increasing the price of a certain good k, the quantity demanded of at least one good of the bundle must decrease.

Proof: Engel aggregation.

- By walras's law: $y = \mathbf{p}\mathbf{x}(\mathbf{p}, y)$,
- Differentiate both sides with respect to income:

$$
\begin{aligned}
\frac{\partial y}{\partial y} &\equiv \mathbf{p}\frac{\partial \mathbf{x}(\mathbf{p}, y)}{\partial y} \\
1 &\equiv \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial y} \\
1 &\equiv \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial y}\frac{x_i y}{x_i y} \\
1 &\equiv \sum_{i=1}^{n} \frac{p_i x_i}{y}\frac{\partial x_i}{\partial y}\frac{y}{x_i} \\
1 &\equiv \sum_{i=1}^{n} s_i \eta_i
\end{aligned}
$$

□

Proof: Cournot aggregation.

- By walras's law: $y = \mathbf{px}(\mathbf{p}, y)$. Now, differentiate both sides with respect to p_j:

$$
\begin{aligned}
\frac{\partial y}{\partial p_j} &\equiv \mathbf{p}\frac{\partial \mathbf{x}(\mathbf{p}, y)}{\partial p_j} \\
0 &\equiv \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} \\
0 &\equiv \left(\sum_{i \neq j} p_i \frac{\partial x_i}{\partial p_j} \right) + \left(\frac{\partial p_j}{\partial p_j} x_j + p_j \frac{\partial x_j}{\partial p_j} \right) \\
-x_j &\equiv \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} \\
-x_j \frac{p_j}{y} &\equiv \sum_{i=1}^{n} p_i \frac{\partial x_i}{\partial p_j} \frac{p_j}{y} \\
-\frac{p_j x_j}{y} &\equiv \sum_{i=1}^{n} \frac{p_i x_i}{y} \frac{\partial x_i}{\partial p_j} \frac{p_j}{x_i}
\end{aligned}
$$

□

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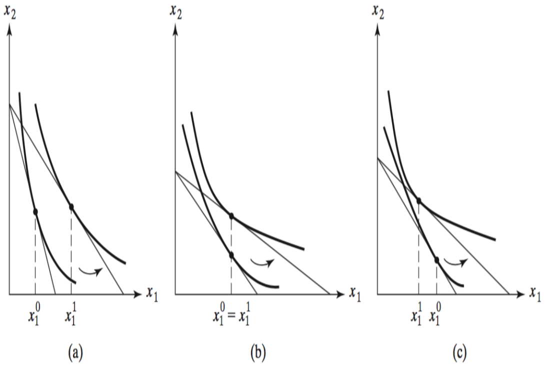
Income and substitution effects

How does quantity demanded change when prices changes?

Income and substitution effects

Figure: Response of quantity demanded to a change in price

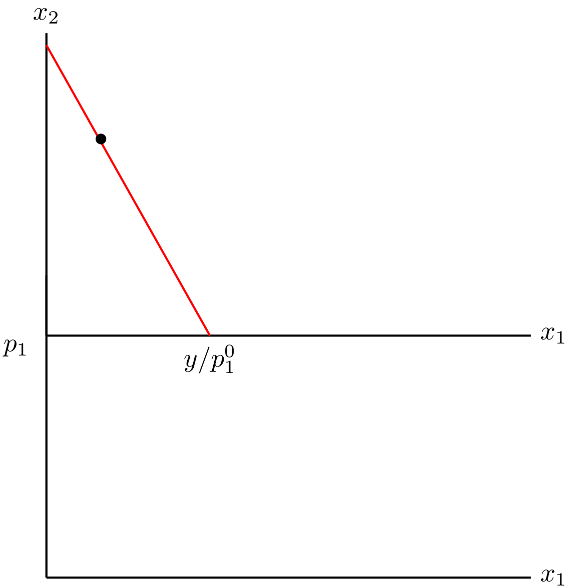


What does the theory predicts?

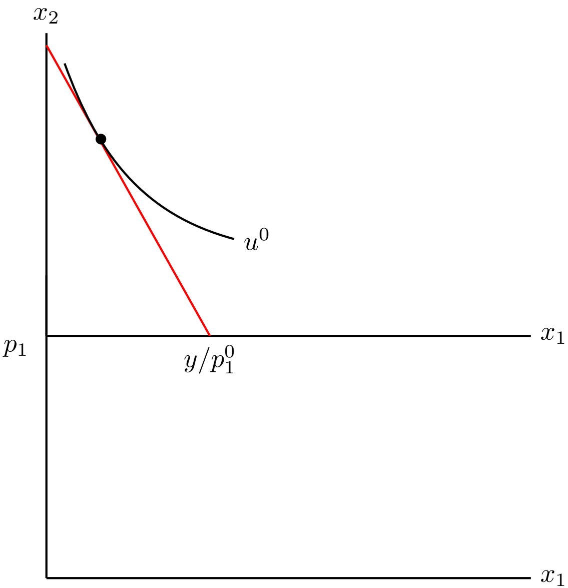
- Price of good declines $\implies$ change in the quantity demanded.
- Substitution effect (SE): We would expect consumer to substitute the relatively cheaper good for the now relatively more expensive ones.
- Income effect (IE): When the price of any one good declines, the consumer's total command over all goods is effectively increased (purchasing power), allowing him to change this purchases of all goods in any way he sees fit

The Hicksian decomposition

- Initial Observation: the consumer achieves some level of utility at the original prices before any change has occurred.
- SE: hypothetical change in consumption that would occur if relative prices were to change to their new levels, but the maximum utility the consumer can achieve were kept the same as before the prices change.
- IE: whatever is left of the total effect after the subtitution effect.



x_2
x_1
p_1
y/p_1^0
x_1



x_2
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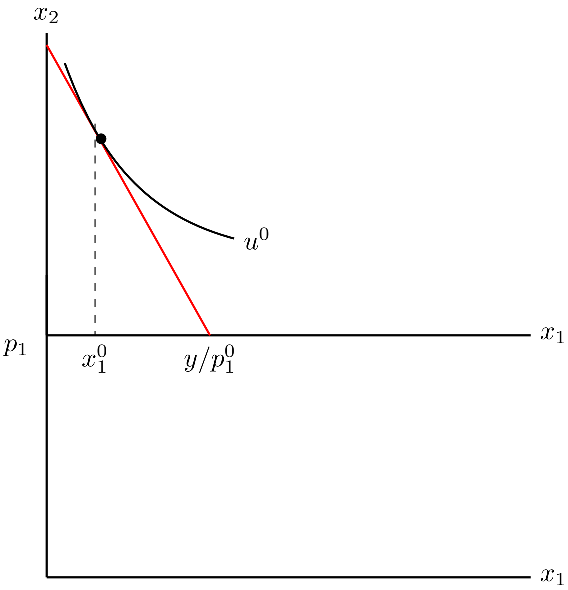
x_1



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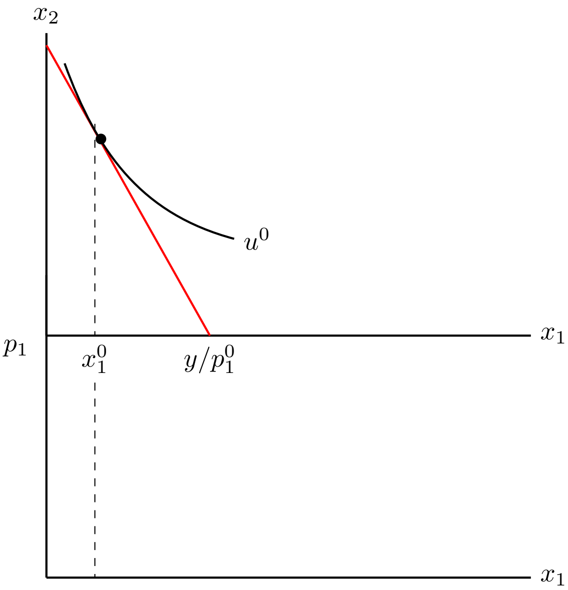
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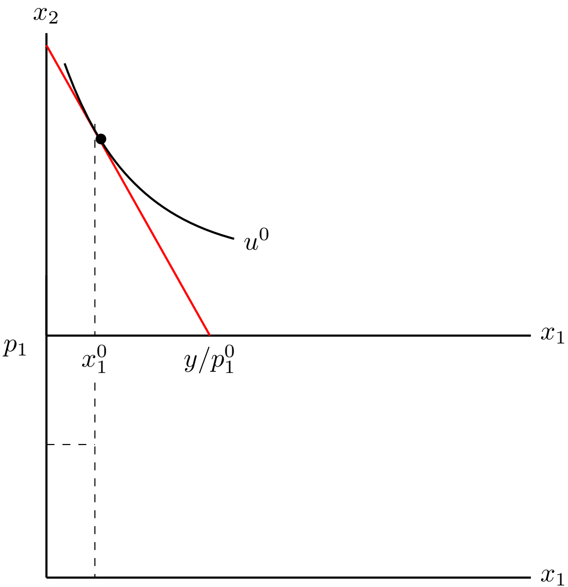

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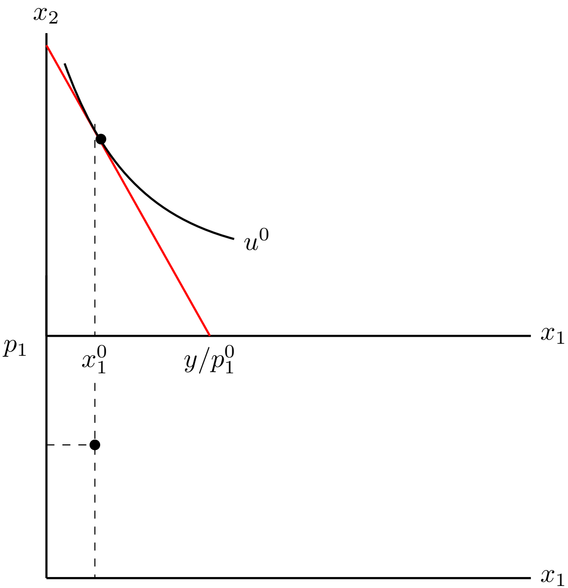

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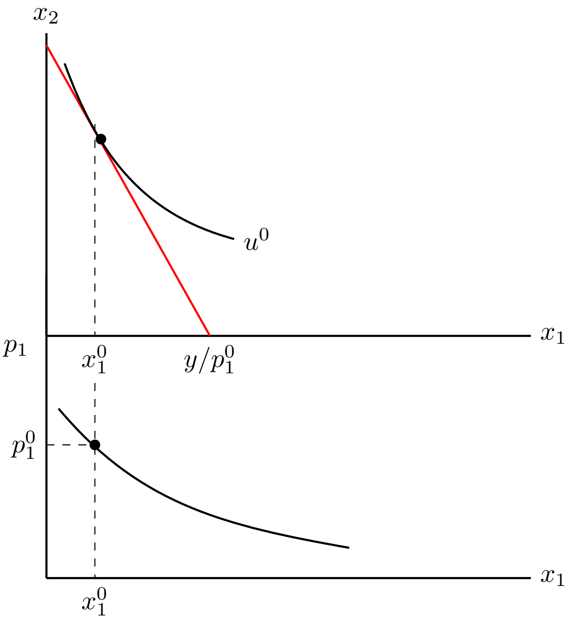
y/p1 0
x1


x_2
u^0
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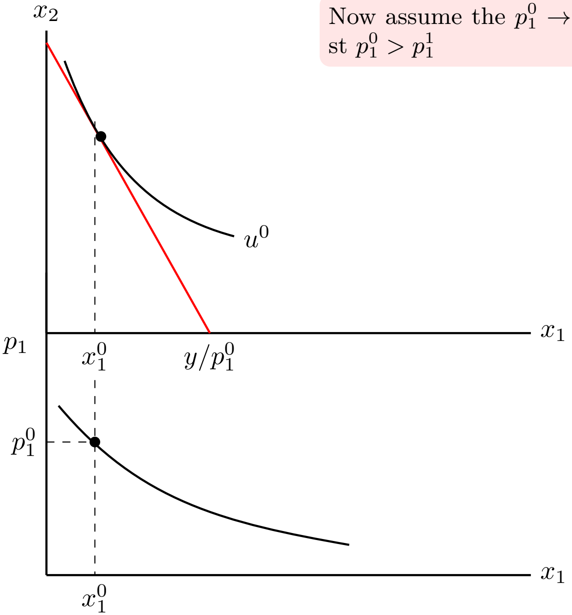
y/p_1^0
p_1^0
x_1
x_1^0

Now assume the $p_1^0 \to p_1^1$ st $p_1^0 > p_1^1$



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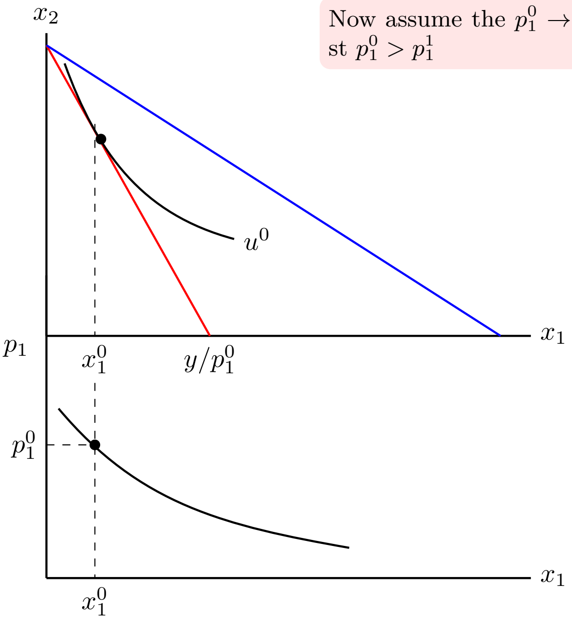

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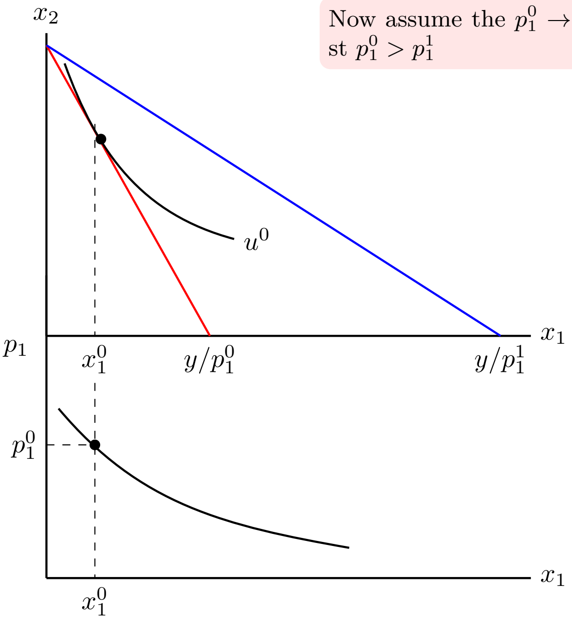

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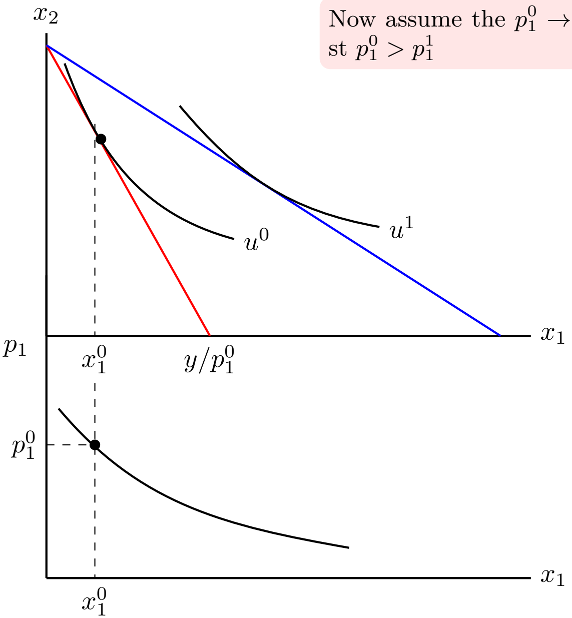

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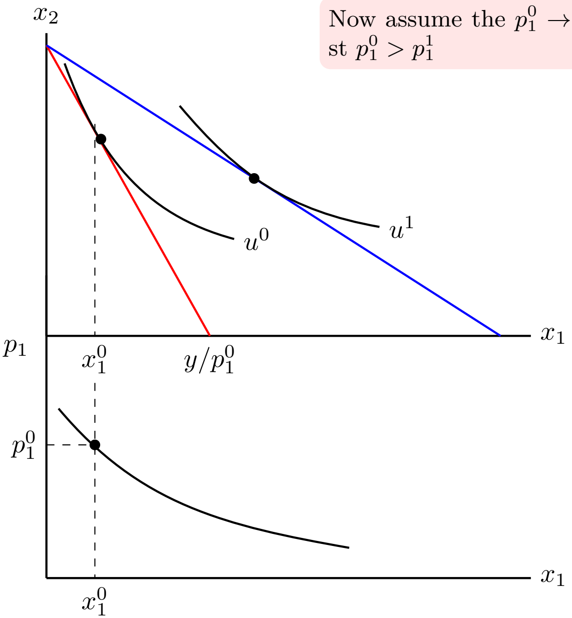

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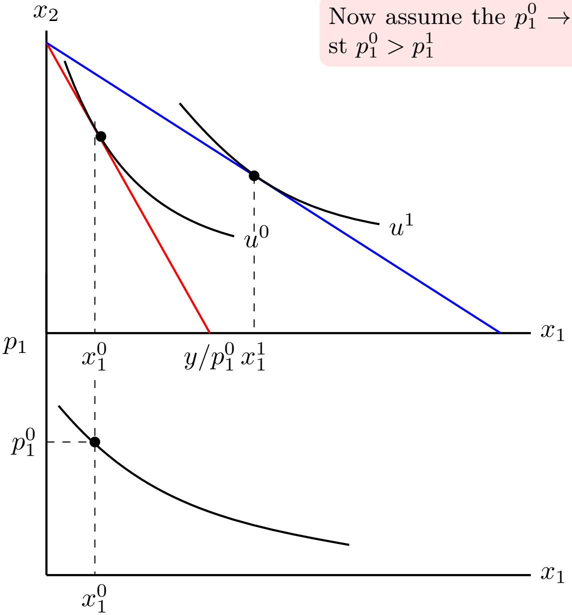

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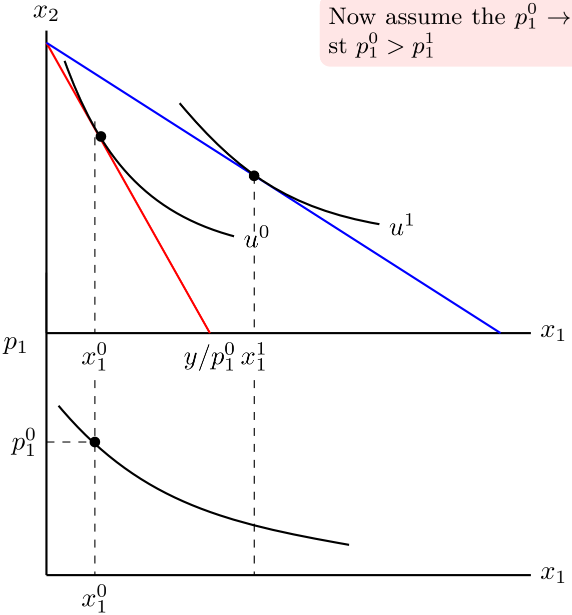

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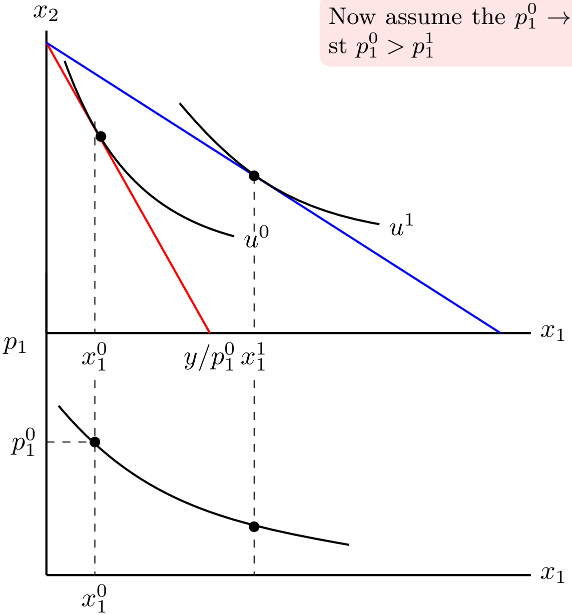

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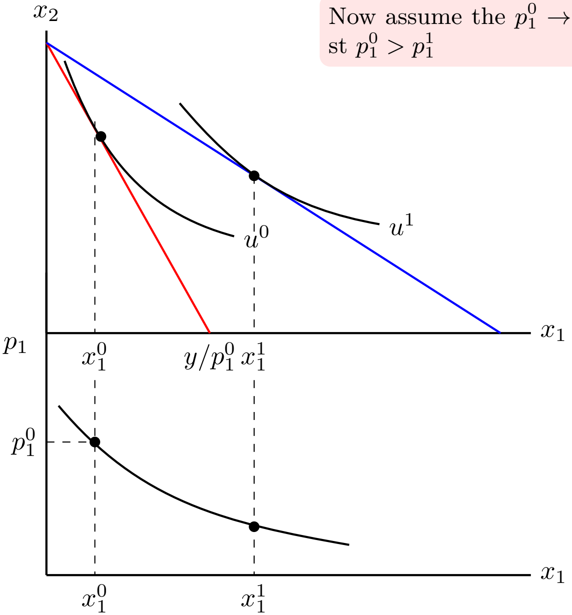

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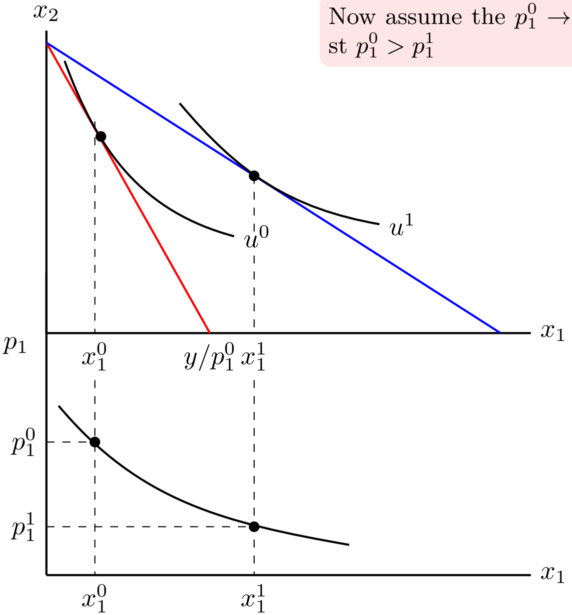
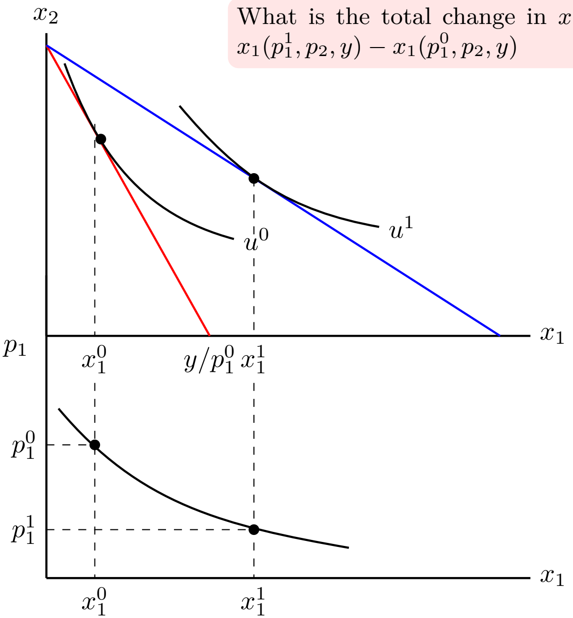


What is the total change in x_1?: $\Delta x_1 = x_1(p_1^1, p_2, y) - x_1(p_1^0, p_2, y)$



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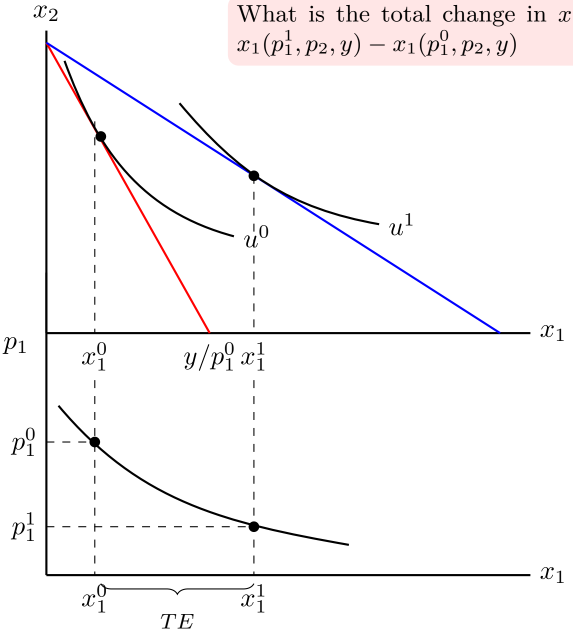
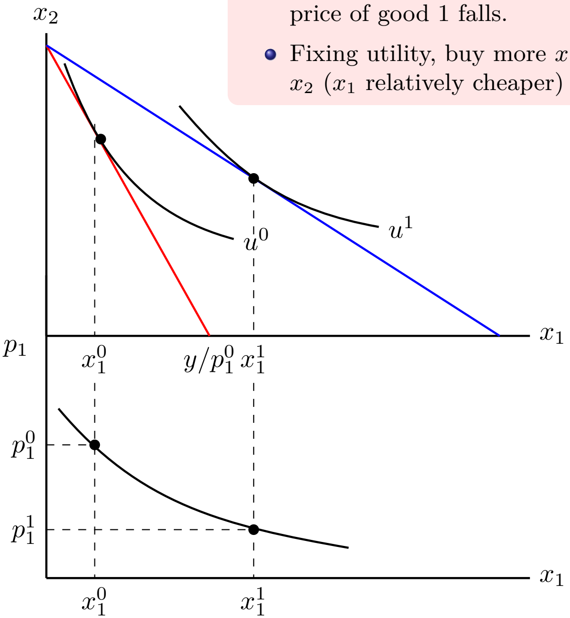
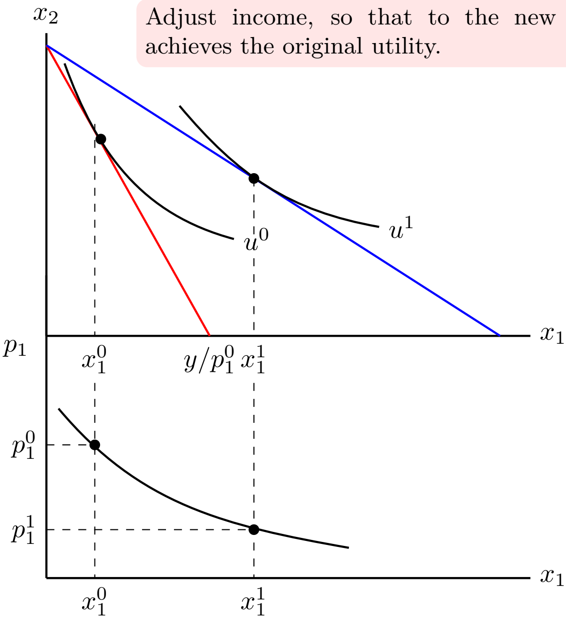
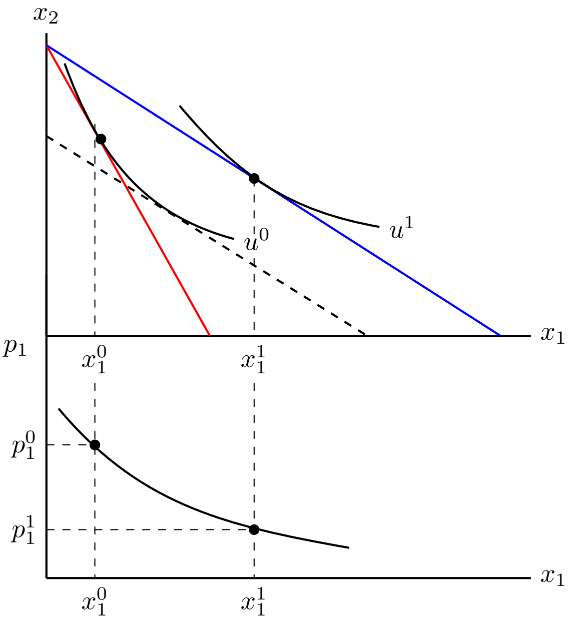


- Note that when p_1 falls, the relative price of good 1 falls.
- Fixing utility, buy more x_1 and less x_2 (x_1 relatively cheaper)



Adjust income, so that to the new relative prices, she achieves the original utility.





x_2
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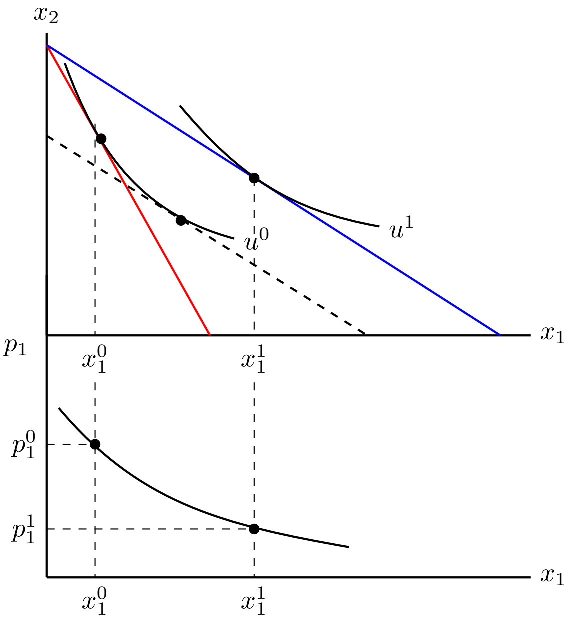

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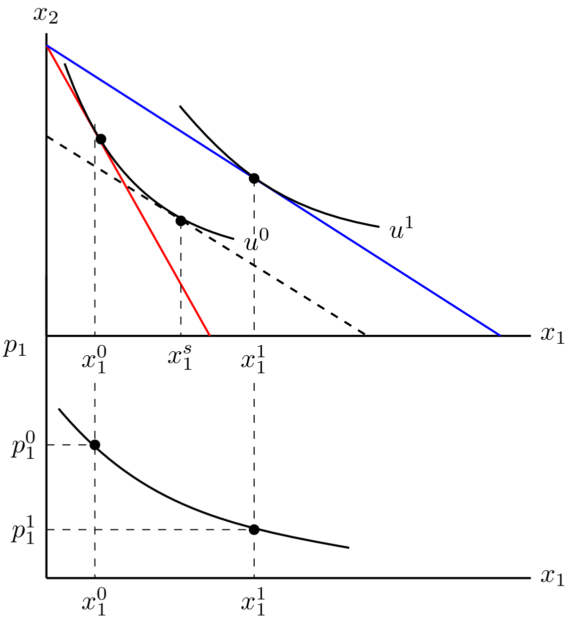
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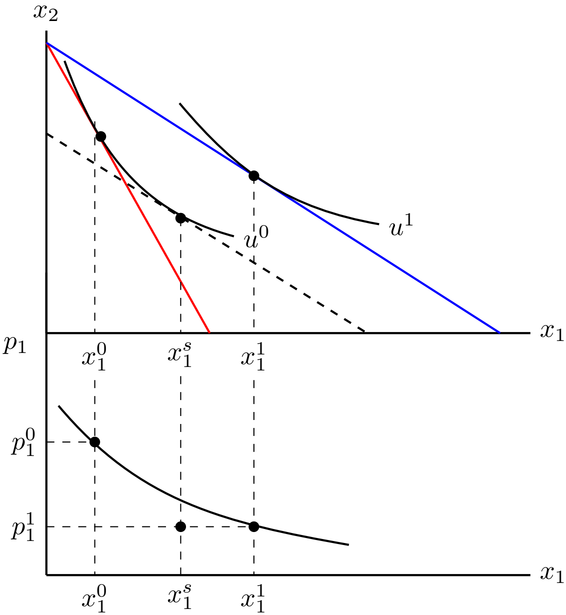
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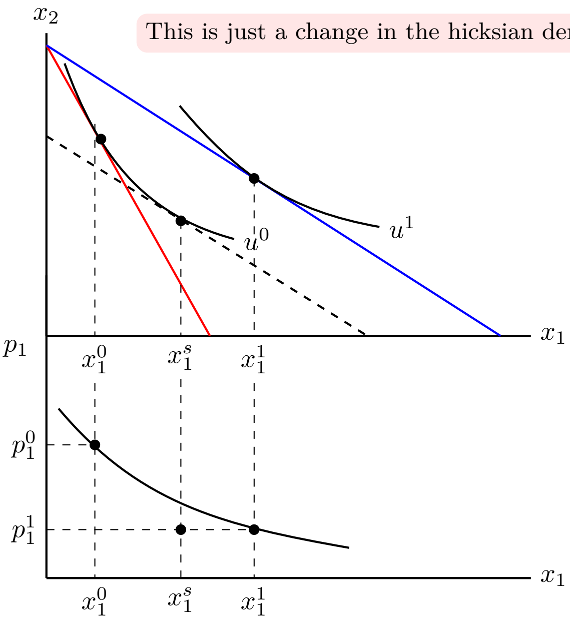
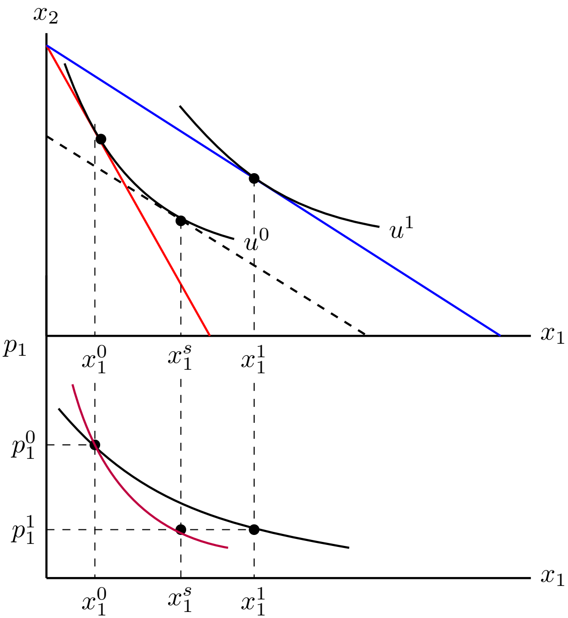
x_1^1

This is just a change in the hicksian demand!





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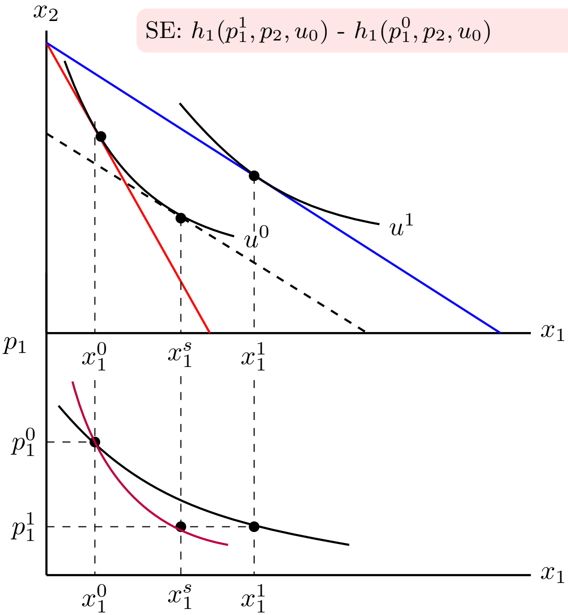
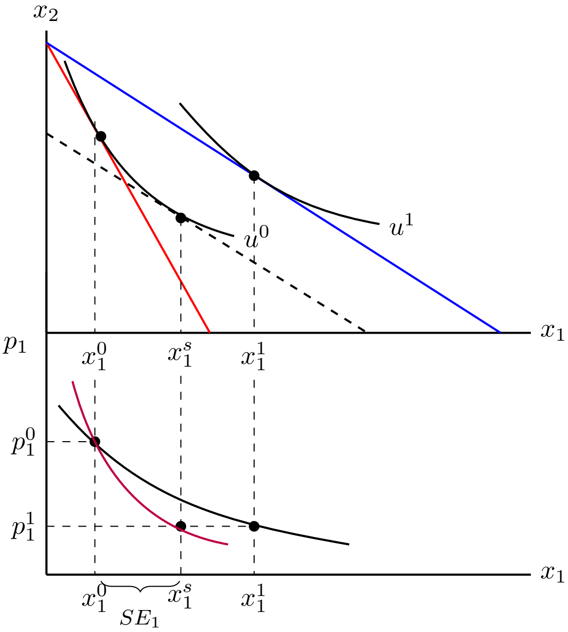
x_1^s
x_1^1
p_1^0
p_1^1
x_1
x_1^0
x_1^s
x_1^1

SE: $h_1(p_1^1, p_2, u_0)$ - $h_1(p_1^0, p_2, u_0)$




x_2
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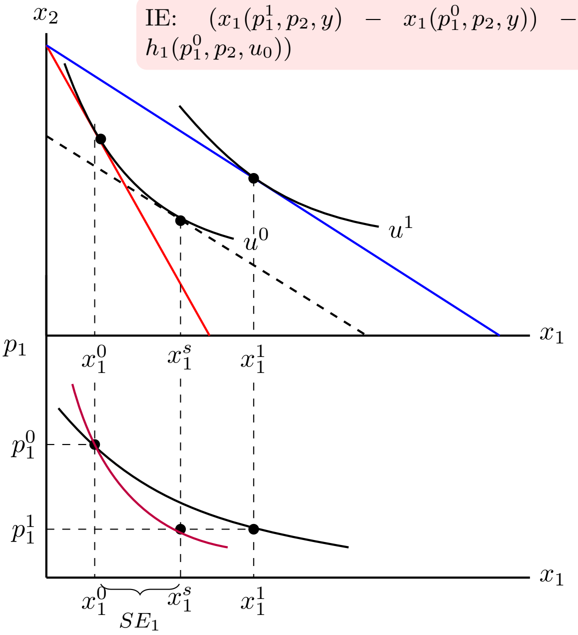
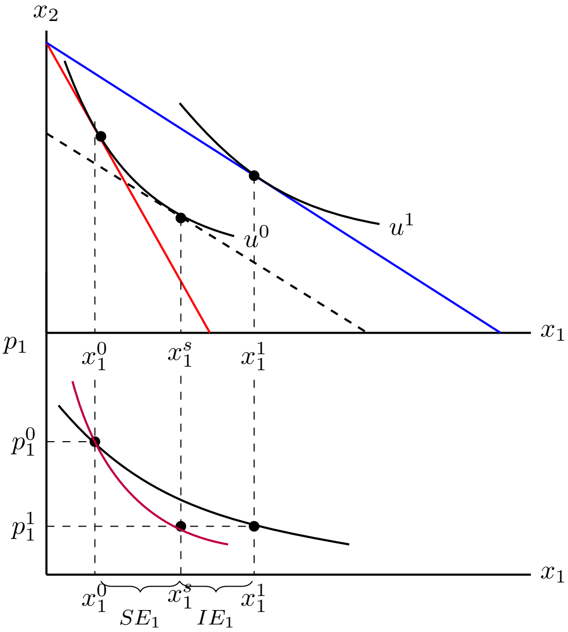
x_1^s
x_1^1
p_1^0
p_1^1
x_1
x_1^0
x_1^s
x_1^1
SE_1

IE: $(x_1(p_1^1, p_2, y) - x_1(p_1^0, p_2, y)) - (h_1(p_1^1, p_2, u_0) - h_1(p_1^0, p_2, u_0))$




x_2
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x_1^s
x_1^1
p_1^0
p_1^1
x_1
x_1^0
x_1^s
x_1^1
SE_1
IE_1

The Slutsky Equation

Theorem (The Slutsky Equation)

Suppose that $u(\cdot)$ is a continuous utility function representing a locally nonsatiated and strictly convex preference relation $\succcurlyeq$ defined on the consumption set $X = \mathbb{R}^n_+$. Then for all $(\mathbf{p}, y)$, and $u = v(\mathbf{p}, y)$

$$\underbrace{\frac{\partial x_i(\mathbf{p}, y)}{\partial p_j}}_{TE} = \underbrace{\frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_j}}_{SE} - \underbrace{x_j(\mathbf{p}, y)\frac{\partial x_i(\mathbf{p}, y)}{\partial y}}_{IE}, \quad i, j = 1, ..., n.$$

or in matrix notation:

$$\underset{n\times n}{\mathbf{D_p x}(\mathbf{p}, y)} = \underset{n\times n}{\mathbf{D_p} x_i^h(\mathbf{p}, u)} - \underset{n\times 1}{\mathbf{D}_y \mathbf{x}(\mathbf{p}, y)} \underset{1\times n}{\mathbf{x}(\mathbf{p}, y)}^\top$$

or in elasticities terms:

$$\epsilon_{ij} \equiv \epsilon_{ij}^h - s_j \eta_i$$

Proof 1.

- Consider $(\bar{\mathbf{p}}, \bar{y})$ and attaining utility level $\bar{u}$.
- By duality $\bar{y} = e(\bar{p}, \bar{y})$,
- By duality we know that $x_n^h(\mathbf{p}, u) = x_n(\mathbf{p}, e(\mathbf{p}, u))\ \forall(\mathbf{p}, u)$.
- Differentiating this expression with respect to p_k and evaluating it at $(\bar{\mathbf{p}}, \bar{y})$ we get (Use Chain Rule):

$$\frac{\partial x_n^h(\bar{\mathbf{p}}, \bar{u})}{\partial p_k} = \frac{\partial x_n(\bar{\mathbf{p}}, e(\bar{\mathbf{p}}, \bar{u}))}{\partial p_k} + \frac{\partial x_n(\bar{\mathbf{p}}, e(\bar{\mathbf{p}}, \bar{u}))}{\partial y} \frac{\partial e(\bar{\mathbf{p}}, \bar{u})}{\partial p_k}$$

- By property of the Hicksian demands: $\frac{\partial e(\bar{\mathbf{p}}, \bar{u})}{\partial p_k} = x_n^h(\bar{\mathbf{p}}, \bar{u})$.
- Since $\bar{y} = e(\bar{p}, \bar{y})$, and $x_n^h(\bar{\mathbf{p}}, \bar{u}) = x_n(\bar{\mathbf{p}}, e(\bar{\mathbf{p}}, \bar{u})) = x_n(\bar{\mathbf{p}}, \bar{u})$, then:

$$\frac{\partial x_n^h(\bar{\mathbf{p}}, \bar{u})}{\partial p_k} = \frac{\partial x_n(\bar{\mathbf{p}}, \bar{y})}{\partial p_k} + \frac{\partial x_n(\bar{\mathbf{p}}, \bar{y})}{\partial y} x_n(\bar{\mathbf{p}}, \bar{y})$$

□

The Slutsky Equation

- The only thing we have done so far is decompose an observable total effect into (1) and observable income effect and (2) and unobservable substitution effect.
- Consider the special case of an own-price change.

$$\frac{\partial x_i(\mathbf{p},y)}{\partial p_i} = \frac{\partial x_i^h(\mathbf{p},u^*)}{\partial p_i} - x_i(\mathbf{p},y)\frac{\partial x_i(\mathbf{p},y)}{\partial y}$$

- Slope of the Marshallian demand curve for good i

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- Slope of the Marshallian demand curve for good i
- Slope of a Hicksian demand curve (not observable)

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- Slope of the Marshallian demand curve for good i
- Slope of a Hicksian demand curve (not observable)
- What can we know about Hicksian demands curves when we cannot even see them?

The Slutsky Equation

- The only thing we have done so far is decompose an observable total effect into (1) and observable income effect and (2) and unobservable substitution effect.
- Consider the special case of an own-price change.

$$\frac{\partial x_i(\mathbf{p},y)}{\partial p_i} = \frac{\partial x_i^h(\mathbf{p},u^*)}{\partial p_i} - x_i(\mathbf{p},y)\frac{\partial x_i(\mathbf{p},y)}{\partial y}$$

- Slope of the Marshallian demand curve for good i
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The Slutsky Equation

Theorem (Negative Own-Substitution Terms)

Let $x_i^h(\mathbf{p}, u)$ be the Hicksian demand for good i. Then

$$\frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_i} \leq 0, \quad i = 1, ..., n.$$

Proof.

Sketch of proof:

1. Use Shephard's lemma.
2. Differentiate Shephard's lemma.
3. Use concavity of $e(\mathbf{p}, u)$.

□

The Slutsky Equation

Theorem (The law of Demand)

A decrease in the own price of a normal good will cause quantity demanded to increase. If an own price decrease causes a decrease in quantity demanded, the good must be inferior.

Sign of the Own Total Effect

	Own SE	Own IE	Own TE
	$\frac{\partial x_i^h(\mathbf{p},u^0)}{\partial p_i}$	$-x_i^* \frac{\partial x_i(\mathbf{p},y)}{\partial y}$	$\frac{\partial x_i(\mathbf{p},y)}{\partial p_i}$
Normal: $\frac{\partial x_i(\mathbf{p},y)}{\partial y} > 0$	≤ 0	≤ 0	≤ 0
$\frac{\partial x_i(\mathbf{p},y)}{\partial y} = 0$	≤ 0	0	≤ 0
Inferior: $\frac{\partial x_i(\mathbf{p},y)}{\partial y} < 0$	≤ 0	≥ 0	≤ 0 $\quad \lvert OSE \rvert > \lvert OIE \rvert$ > 0 $\quad$ Giffen: $\lvert OSE \rvert < \lvert OIE \rvert$

The Slutsky Equation

Theorem (Symmetric Substitution Terms)

Let $\mathbf{x}^h(\mathbf{p}, u)$ *be the consumer's system of Hicksian demands and suppose that* $e(\cdot)$ *is twice continuously differentiable. Then,*

$$\frac{\partial x_i^h(\mathbf{p}, u)}{\partial p_j} = \frac{\partial x_j^h(\mathbf{p}, u)}{\partial p_i}, \quad i, j = 1, ..., n.$$

This is the famous Young's Theorem in calculus.

The Slutsky Equation

Theorem (Negative Semidefinite Substitution Matrix)

Let $\mathbf{x}^h(\mathbf{p}, u)$ be the consumer's system of Hicksian demands, and let

$$\sigma(\mathbf{p}, u) \equiv \begin{pmatrix} \frac{\partial x_1^h(\mathbf{p},u)}{\partial p_1} & \cdots & \frac{\partial x_1^h(\mathbf{p},u)}{\partial p_n} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_n^h(\mathbf{p},u)}{\partial p_1} & \cdots & \frac{\partial x_n^h(\mathbf{p},u)}{\partial p_n} \end{pmatrix}$$

called the ***substitution matrix****, contain all the Hicksian substitution terms. Then the matrix $\sigma(\mathbf{p}, u)$ is negative semidefinite.*

This is the Hessian Matrix of the expenditure function $(\partial x_j^h(\mathbf{p}, u^0)/\partial p_i) \equiv \partial^2 e(\mathbf{p}, u^0)/\partial p_j \partial p_i)$, which is concave in $\mathbf{p}$. (That is why $\sigma(\mathbf{p}, u)$ is negative semidefinite). The diagonal elements are no-positive.

Singularity

This implies that: $\mathbf{p}\boldsymbol{\sigma}(\mathbf{p}, u) = \mathbf{0}$, that is

$$\mathbf{p}\boldsymbol{\sigma}(\mathbf{p}, u) = \begin{pmatrix} p_1 & p_2 & p_3 \end{pmatrix} \begin{pmatrix} \sigma_{11} & \sigma_{12} & \sigma_{13} \\ \sigma_{21} & \sigma_{22} & \sigma_{23} \\ \sigma_{31} & \sigma_{32} & \sigma_{33} \end{pmatrix}$$

give us:

$$\begin{aligned} p_1\sigma_{11} + p_2\sigma_{21} + p_3\sigma_{31} &= 0 \\ p_1\sigma_{12} + p_2\sigma_{22} + p_3\sigma_{32} &= 0 \\ p_1\sigma_{13} + p_2\sigma_{23} + p_3\sigma_{33} &= 0 \end{aligned}$$

Example

Let $x_i^* = x_i(p_1, p_2, p_3, y); \forall i = 1, 2, 3$. We observe $x_1^* = 1, x_2^* = 2, x_3^* = 1$, when $p_1 = p_2 = p_3 = 2$. Furthermore, we know that:

$$\frac{\partial x_2}{\partial y} = 2; \frac{\partial x_3}{\partial y} = 2; \frac{\partial x_2}{\partial p_2} = -10; \frac{\partial x_3}{\partial p_3} = -2; \frac{\partial x_3}{\partial p_1} = 4$$

Find $\sigma(\mathbf{p}, u)$.

- Use Slustky Equation:

$$\frac{\partial x_i(\mathbf{p}, y)}{\partial p_j} = \underbrace{\frac{\partial x_i^h(\mathbf{p}, u^*)}{\partial p_j}}_{\sigma_{ij}} - x_j(\mathbf{p}, y)\frac{\partial x_i(\mathbf{p}, y)}{\partial y} \implies \sigma_{ij} = \frac{\partial x_i(\mathbf{p}, y)}{\partial p_j} + x_j^* \frac{\partial x_i(\mathbf{p}, y)}{\partial y}$$

So:

$$\begin{aligned}
\sigma_{22} &= \frac{x_2}{p_2} + x_2^* \frac{\partial x_2}{\partial y} = -10 + 2.2 \implies s_{22} = -6 \\
\sigma_{31} &= \frac{x_3}{p_1} + x_1^* \frac{\partial x_3}{\partial y} = 4 + 1.2 \implies s_{31} = 6 \\
\sigma_{23} &= \frac{x_2}{p_3} + x_3^* \frac{\partial x_2}{\partial y} = -2 + 1.2 \implies s_{23} = 0
\end{aligned}$$

Example

- Use symmetric property: $\sigma_{13} = \sigma_{31} = 6$ and $\sigma_{32} = \sigma_{23} = 0$
- Use singularity
  - $p_1\sigma_{12} + p_2\sigma_{22} + p_3\sigma_{32} = 0 \implies \sigma_{12} = 6 = \sigma_{21}$,
  - $p_1\sigma_{11} + p_2\sigma_{21} + p_3\sigma_{31} \implies \sigma_{11} = -12$
  - $p_1\sigma_{13} + p_2\sigma_{23} + p_3\sigma_{33} \implies \sigma_{33} = -6$

So,

$$\sigma(\mathbf{p}, u) \equiv \begin{pmatrix} -12 & 6 & 6 \\ 6 & -6 & 0 \\ 6 & 0 & -6 \end{pmatrix}$$

The Slutsky Equation

Theorem (Symmetric and Negative Slutsky Matrix)

Let $\mathbf{x}(\mathbf{p}, y)$ *be the consumer's Marshallian demand system. Define the* ij*th Slustsky term as*

$$\frac{\partial x_i(\mathbf{p}, y)}{\partial p_j} + x_j(\mathbf{p}, y)\frac{\partial x_i(\mathbf{p}, y)}{\partial y}$$

and form the entire $n \times n$ ***Slustky matrix*** *of price and income responses as follows:*

$$\mathbf{S}(\mathbf{p}, y) = \begin{pmatrix} \frac{\partial x_1(\mathbf{p},y)}{\partial p_1} + x_1(\mathbf{p}, y)\frac{\partial x_1(\mathbf{p},y)}{\partial y} & \dots & \frac{\partial x_1(\mathbf{p},y)}{\partial p_n} + x_n(\mathbf{p}, y)\frac{\partial x_1(\mathbf{p},y)}{\partial y} \\ \vdots & \ddots & \vdots \\ \frac{\partial x_n(\mathbf{p},y)}{\partial p_1} + x_1(\mathbf{p}, y)\frac{\partial x_n(\mathbf{p},y)}{\partial y} & \dots & \frac{\partial x_n(\mathbf{p},y)}{\partial p_n} + x_n(\mathbf{p}, y)\frac{\partial x_n(\mathbf{p},y)}{\partial y} \end{pmatrix}$$

Then $\mathbf{S}(\mathbf{p}, y)$ *is symmetric and negative semidefinite.*

Last Remarks

Implications

- Homogeneity: Demand must respond to an overall, equiproportionate change in all prices and income simultaneously.
- Slutsky equation: Give us qualitative information or “sign restrictions” on how the system of demand functions must respond to very general kind of price changes.
- Aggregation: Information on how the quantities demanded must all “hang together” across the system of demand functions.