FEN, Universidad de Talca

Spatial Econometrics: Problem Set 2

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September, 2017

1 Theory

1. (Moments of the SAC Model) Consider the following SAC model:

$$\begin{aligned}\mathbf{y} &= \rho\mathbf{W}\mathbf{y} + \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \\ \mathbf{u} &= \lambda\mathbf{M}\mathbf{u} + \boldsymbol{\varepsilon} \\ \boldsymbol{\varepsilon} &\sim \mathrm{N}(\mathbf{0}, \mathbf{I}_n\sigma^2_\epsilon)\end{aligned} \tag{1.1}$$

   Derive $\mathbb{E}(\mathbf{y}|\mathbf{W},\mathbf{M},\mathbf{X})$, $\mathrm{Var}(\mathbf{y}|\mathbf{W},\mathbf{M},\mathbf{X})$, $\mathbb{E}(\mathbf{u}|\mathbf{W},\mathbf{M},\mathbf{X})$ and $\mathrm{Var}(\mathbf{u}|\mathbf{W},\mathbf{M},\mathbf{X})$. What do you conclude?

2. (Singularity) Show the following. If $\mathbf{W}$ is row normalized, $(\mathbf{I}_n - \rho\mathbf{W})$ is singular at $\rho = 1$.

3. (Marginal Effects) Consider the following model:

$$\mathbf{y} = \mathbf{X}\boldsymbol{\beta} + \rho_1\mathbf{W}_1\mathbf{y} + \rho_2\mathbf{W}_2\mathbf{y} + \boldsymbol{\varepsilon}$$

   where $\mathbf{W}_1$ and $\mathbf{W}_2$ are weighting matrices. Assuming typical conditions, give expression for the marginal effects.

4. (Direct and indirect effects in spatial models) Assume 3 regions with the following row-normalized spatial weight matrix

$$\mathbf{W}^s = \begin{pmatrix} 0 & 1 & 0 \\ w_{21} & 0 & w_{23} \\ 0 & 1 & 0 \end{pmatrix} \tag{1.2}$$

   Derive the Total Direct and Total Indirect Effects for the following models:

a) Spatial Durbin Model given by:

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \alpha \boldsymbol{\imath}_n + \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \boldsymbol{\varepsilon} \tag{1.3}$$

b) Spatial Lag Model given by:

$$\mathbf{y} = \rho \mathbf{W}\mathbf{y} + \alpha \boldsymbol{\imath}_n + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1.4}$$

c) Spatial Durbin Error Model given by:

$$\mathbf{y} = \alpha \boldsymbol{\imath}_n + \mathbf{X}\boldsymbol{\beta} + \mathbf{W}\mathbf{X}\boldsymbol{\theta} + \mathbf{u} \tag{1.5}$$

$$\mathbf{u} = \lambda \mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon} \tag{1.6}$$

d) OLS given by:

$$\mathbf{y} = \alpha \boldsymbol{\imath}_n + \mathbf{X}\boldsymbol{\beta} + \boldsymbol{\varepsilon} \tag{1.7}$$

e) Spatial Error model given by:

$$\begin{aligned} \mathbf{y} &= \alpha \boldsymbol{\imath}_n + \mathbf{X}\boldsymbol{\beta} + \mathbf{u} \\ \mathbf{u} &= \lambda \mathbf{W}\mathbf{u} + \boldsymbol{\varepsilon} \end{aligned} \tag{1.8}$$

5. (Marginal Effects) Consider your results for the SLM and SDM models from previous question. Show that for the SLM model the ratio between the indirect and the direct effect of a particular explanatory variable is independent of β_k. Show that this is not the case for the SDM model. What do you conclude?

6. (Sign of Total Effects) Recall that if the row-sums of $\mathbf{W}$ is less than or equal to one and ρ is in the proper parameter space, i.e., $\rho < 1$, the total average effect for variable r can be computed as $\beta_r/(1-\rho)$. What is the sign of the parameter that matters the most when calculating the sign of the total effect? Does the ρ or β_r?