Universidad Católica del Norte, Economic Department

Econometrics: Problem Set 1

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This homework is due on 12th, April at 18:00 hrs. Please, write your answers clearly.

1 Theory

1. (Expectation) Let $X_i \sim \mathrm{N}(0,1), i = 1, 2$ be two independent random variables and $h(X_1, X_2) = X_1^2 + X_2^2$. Find $\mathbb{E}(h(X_1, X_2))$.

2. (Strict Exogeneity) Let the model be $y = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \epsilon$. What would be the theoretical implication if we assume that $\mathbb{E}(\epsilon) = 0, \mathrm{Cov}(x_1, \epsilon) = 0$ and $\mathrm{Cov}(x_2, \epsilon) = 0$, instead of assuming that $\mathbb{E}(\epsilon|x_1, x_2) = 0$?

3. (LIE) Show and give an example of the following LIE: If $\mathbb{E}\,|y| < \infty$, then for any random vector $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$,

$$\mathbb{E}\left[\mathbb{E}(y|\boldsymbol{x}_1, \boldsymbol{x}_2)|\boldsymbol{x}_1\right] = \mathbb{E}(y|\boldsymbol{x}_1).$$

4. (Partial effects) Consider the following model assuming that y, x_1 and x_2 are random variables:

$$\mathbb{E}(y|x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 x_1 x_2$$

   a) Find the partial effects of x_1 and x_2 on $\mathbb{E}(y|x_1, x_2)$.

   b) If we write the equation as:

$$y = \beta_0 + \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_2^2 + \beta_4 x_1 x_2 + \epsilon,$$

what can said about $\mathbb{E}(\epsilon|x_1, x_2)$? What about $\mathbb{E}(\epsilon|x_1, x_2, x_2^2, x_1x_2)$?

c) In the equation of part (b), what can be said about $\text{Var}(\epsilon|x_1, x_2)$?

d)

5. (Partial Effects) Suppose that:

$$\mathbb{E}(y|x_1, x_2) = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_1 x_2$$

Suppose also that x_1 and x_2 have zero means. Show that β_1 is the expected value of $\partial\mathbb{E}(y|x_1, x_2)/\partial x_1$ (where the expectation is across the population distribution of x_2). Provide a similar interpretation for β_2.

6. (More variables implies less variance of the error term) Consider the following two models:

$$y = \mu_1(\boldsymbol{x}, \boldsymbol{z}) + \epsilon_1, \quad \mathbb{E}(\epsilon_1|\boldsymbol{x}, \boldsymbol{z}) = 0$$
$$y = \mu_2(\boldsymbol{x}) + \epsilon_2, \quad \mathbb{E}(\epsilon_2|\boldsymbol{x}) = 0$$

Assuming that $\text{Var}(y|\boldsymbol{x}, \boldsymbol{z})$ and $\text{Var}(y|\boldsymbol{x})$ are both constant, what can you say about the relationship between $\text{Var}(\epsilon_1)$ and $\text{Var}(\epsilon_2)$? (Hint: use the fact that $\mathbb{E}[\text{Var}(y|\boldsymbol{x})] \geq \mathbb{E}[\text{Var}(y|\boldsymbol{x}, \boldsymbol{z})]$)

7. (Elemental Matrix Identities for OLS) Show the following:

a) $\widehat{\boldsymbol{y}} = \boldsymbol{P}\boldsymbol{y}$, $\widehat{\boldsymbol{\varepsilon}} = \boldsymbol{M}\boldsymbol{y} = \boldsymbol{M}\boldsymbol{\varepsilon}$.

b) $\text{SSR} = \boldsymbol{\varepsilon}'\boldsymbol{M}\boldsymbol{\varepsilon}$