UNIVERSIDAD CATÓLICA DEL NORTE, ECONOMIC DEPARTMENT

Econometrics: Problem Set 1

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This homework is due on 29th, March at 18:00 hrs. Please, write your answers clearly.

1 THEORY

1. (LIE) Show and give an example of the following LIE: If $\mathbb{E}\left|y\right| < \infty$, then for any random vector $\boldsymbol{x}_1$ and $\boldsymbol{x}_2$,

$$\mathbb{E}\left[\mathbb{E}(y|\boldsymbol{x}_1, \boldsymbol{x}_2)|\boldsymbol{x}_1\right] = \mathbb{E}(y|\boldsymbol{x}_1).$$

2. (Elemental Matrix Identities for OLS) Show the following:
   a) $\widehat{\boldsymbol{y}} = \boldsymbol{P}\boldsymbol{y}$, $\widehat{\boldsymbol{\varepsilon}} = \boldsymbol{M}\boldsymbol{y} = \boldsymbol{M}\boldsymbol{\varepsilon}$.
   b) $\text{SSR} = \boldsymbol{\varepsilon}'\boldsymbol{M}\boldsymbol{\varepsilon}$

3. (Conditional Variance) Let Y and X be two random variables on the same probability space, and the variance of Y to be finite. Show that:

$$\text{Var}(Y) = \mathbb{E}\left[\text{Var}(Y|X)\right] + \text{Var}\left[\mathbb{E}(Y|X)\right]$$

4. (More variables implies less variance of the error term) Consider the following two models:

$$\begin{aligned} y &= \mu_1(\boldsymbol{x}, \boldsymbol{z}) + \epsilon_1, \quad \mathbb{E}(\epsilon_1|\boldsymbol{x}, \boldsymbol{z}) = 0 \\ y &= \mu_2(\boldsymbol{x}) + \epsilon_2, \quad \mathbb{E}(\epsilon_2|\boldsymbol{x}) = 0 \end{aligned}$$

Assuming that $\text{Var}(y|\boldsymbol{x}, \boldsymbol{z})$ and $\text{Var}(y|\boldsymbol{x})$ are both constant, what can you say about the relationship between $\text{Var}(\epsilon_1)$ and $\text{Var}(\epsilon_2)$? (Hint: use the fact that $\mathbb{E}[\text{Var}(y|\boldsymbol{x})] \geq \mathbb{E}[\text{Var}(y|\boldsymbol{x}, \boldsymbol{z})]$)

5. (Regression without a constant) Consider the simple regression model $y_i = \beta_0 + \beta_1 x_i + \epsilon_i$ under the assumption of linearity, strict exogeneity, no multicolineality, and homokedasticity.

   a) Show that $\widehat{\beta}_1$—the OLS estimate of β_1—is unbiased.

   b) Let $\widetilde{\beta}_1$ be the estimator of β_1 obtained by assuming the intercept is zero. Find $\mathbb{E}(\widetilde{\beta}_1|x_1, x_2, ..., x_n)$ in terms of x_i, β_0 and β_1. Verify that $\widetilde{\beta}_1$ is unbiased for β_1 when the population intercept is zero. Are there other cases where $\widetilde{\beta}_1$ is unbiased?

   c) Find the variance of $\widetilde{\beta}_1$.

   d) Show that $\text{Var}(\widetilde{\beta}_1) \leq \text{Var}(\widehat{\beta}_1)$.