Universidad Católica del Norte, Economic Department

Microeconomics: Problem Set 1

Professor: Mauricio Sarrias

First semester, 2020

This homework is due on 1st, April at 18:00 hrs. Please, write your answers clearly.

1 Math-Review

1. (Convex sets) The non-negative orthant is a very important set in microeconomic theory. For example, prices are not allowed to be negative. Show that the non-negative orthant is a convex set. Just recall that the non-negative orthant includes zero. (Hint: Show that any line segment joining any two points in $\mathbb{R}^n_+ = \{\mathbf{x} : x_i > 0 \ \ , \forall i = 1, ..., n\}$ is contained in $\mathbb{R}^n_+$.)

2. (Open Sets) Show that intersection of any finite number of open sets is an open set.

3. (Closed sets) Show that the intersection of closed sets is a closed set.

4. (Concavity and convexity, You must read JR's appendix in order to solve this question) Show that the function f is convex if and only if the function $-f$ is concave (Do not assume that the function f is differentiable).

2 Preferences

1. (Consumption set properties) In general, the consumption set is defined in the non-negative orthant. Let $X = \mathbb{R}^2_+$. Verify that X satisfies all five properties required of a consumption set in Assumption 1.1 of R&J Book.

2. (Preference Axioms) Show that $\forall \mathbf{x}^1, \mathbf{x}^2, \mathbf{x}^3 \in X$:

a) $\mathbf{x}^1 \succ \mathbf{x}^2 \succ \mathbf{x}^3 \implies \mathbf{x}^1 \succ \mathbf{x}^3$.

b) $\mathbf{x}^1 \sim \mathbf{x}^2 \succcurlyeq \mathbf{x}^3 \implies \mathbf{x}^1 \succcurlyeq \mathbf{x}^3$.

3. (Preference's Sets) Plot the "at least as good set" set for the following utility functions:

   a) $u(x_1, x_2) = \min\left\{x_1, x_2, \frac{x_1+x_2}{3}\right\}$

   b) $u(x_1, x_2) = \min\{x_1, x_2\} + \max\{x_1, x_2\}$.

   c) $u(x_1, x_2) = \ln x_1 - \ln x_2, \quad x_1, x_2 \neq 0$.

4. (Utility and Preferences): Check that the following preferences relation $\forall \mathbf{x}, \mathbf{x}' \in X \subseteq \mathbb{R}^2_+ : \mathbf{x} \succcurlyeq \mathbf{x}' \iff x_1 \geq x_1' - 1$ satisfy: (i) completeness, (ii) reflexivity[1], (iii) transitivity, (iv) monotonicity and (v) weak convexity. Also derive the different classes of sets and explain what kind of preferences they represent (Hint: good 2 is neutral).

[1] $\succcurlyeq$ is reflexive if $\mathbf{x} \succcurlyeq \mathbf{x}$