UNIVERSIDAD CATÓLICA DEL NORTE, ECONOMIC DEPARTMENT

Spatial Econometrics: Final Exam

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- If you believe a question is unclear, please state how you interpret the question.
- Please, use formal mathematical language wherever possible.
- Please assume whatever you need to assume for the sake of your arguments.
- You must show all work for partial credit to be awarded.
- Total points: 40.

1 QUESTIONS

1. Consider the following spatial structure:

$$\mathbf{y}_n = \rho_0^1 \mathbf{W}_n^1 \mathbf{y}_n + \rho_0^2 \mathbf{W}_n^2 \mathbf{y}_n + \mathbf{X}_n \boldsymbol{\beta}_0 + \boldsymbol{\varepsilon}_n \tag{1}$$

   where $\boldsymbol{\varepsilon}$ has mean $\mathbf{0}$ and variance-covariance matrix $\sigma^2 \mathbf{I}_n$; $\mathbf{W}_n^1$ and $\mathbf{W}_n^2$ are observed exogenous weight matrices.

   a) (5 pts) Suggest and instrumental variable estimation procedure for this model which accounts for the endogeneity of $\mathbf{W}_n^1 \mathbf{y}_n$ and $\mathbf{W}_n^2 \mathbf{y}_n$.

   b) (15 pts) Let $\boldsymbol{\gamma}_0 = (\boldsymbol{\beta}_0', \rho_0^1, \rho_0^2)$. Describe the large sample distribution of your instrumental variable estimator given in part (a).

2. (20 pts) Consider the following spatial structure:

$$\begin{aligned} \mathbf{y}_n &= \mathbf{X}_n \boldsymbol{\beta}_0 + \mathbf{u}_n \\ \mathbf{u}_n &= \rho_0 \mathbf{W}_n \mathbf{u}_n + \lambda_0 \mathbf{W}_n \boldsymbol{\varepsilon}_n + \boldsymbol{\varepsilon}_n \end{aligned} \tag{2}$$

where $\boldsymbol{\varepsilon}$ has mean $\mathbf{0}$ and variance-covariance matrix $\sigma^2\mathbf{I}_n$. Determine equations which could be used in a GMM approach to estimate ρ_0, λ_0 and σ^2.

The following theorem can be useful:

CLT for triangular arrays with homokedastic errors: Let $\{v_{i,n}, 1 \leq i \leq n, n \geq 1\}$ be a triangular array of identically distributed random variables. Assume that the random variables $\{v_{i,n}, 1 \leq i \leq n\}$ are jointly independently distributed for each n with $\mathbb{E}(v_{i,n}) = 0$ and $\mathbb{E}(v_{i,n}^2) = \sigma^2 < \infty$. Let $\{a_{ij,n}, 1 \leq i \leq n, n \geq 1\}, j = 1, ..., k$ be triangular arrays of real numbers that are bounded in absolute value. Further let

$$\mathbf{v}_n = \begin{pmatrix} v_{1,n} \\ \vdots \\ v_{n,n} \end{pmatrix}, \quad \mathbf{A}_n = \begin{pmatrix} a_{11,n} & \dots & a_{1k,n} \\ \vdots & & \vdots \\ a_{n1,n} & \dots & a_{nk,n} \end{pmatrix}$$

Assume that $\lim_{n\to\infty} n^{-1}\mathbf{A}_n^\top \mathbf{A}_n = \mathbf{Q}_{AA}$ is finite and nonsingular matrix. Then

$$\frac{1}{\sqrt{n}}\mathbf{A}_n^\top \mathbf{v}_n \xrightarrow{d} \mathrm{N}(\mathbf{0}, \sigma^2\mathbf{Q}_{AA})$$