Universidad Católica del Norte, Economic Department

Spatial Econometrics: Final Exam

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- If you believe a question is unclear, please state how you interpret the question.
- Please, use formal mathematical language wherever possible.
- Please assume whatever you need to assume for the sake of your arguments.
- You must show all work for partial credit to be awarded.
- Total points: 45.

1 Questions

Consider the following spatial DGP:

$$\begin{aligned} \mathbf{y}_n &= \rho_0 \mathbf{W}_n \mathbf{y}_n + \mathbf{X}_n \boldsymbol{\beta}_0 + \mathbf{u}_n \\ \mathbf{u}_n &= \lambda_0 \mathbf{M}_n \mathbf{u}_n + \boldsymbol{\varepsilon}_n \end{aligned} \tag{1}$$

1. (SLM Model) Assume that $\lambda_0 = 0$, $\mathbb{E}(u_{i,n}) = 0$ and $\mathbb{E}(u_{i,n}^2) = \sigma^2$, which is finite.
   a) (5 pts) If we would like to estimate the parameters using S2SLS, and assuming that $\mathbf{X}_n$ are exogenous, what would be the optimal instruments? What is the intuition behind those instruments?
   b) (5 pts) What are the advantages of disadvantages of estimating S2SLS estimator compare with the ML estimator?
2. (SAC Model) Now assume that $\mathbb{E}(\epsilon_{i,n}) = 0$ and $\mathbb{E}(\epsilon_{i,n}^2) = \sigma^2$.
   a) (10 pts) Explain in detail the procedure to obtain the estimator of λ_0.

b) (5 pts) Explain the procedure to obtain the FGS2SLS estimator.

c) (20 pts) Derive the asymptotic distribution of the FGS2SLS estimator.

The following theorem can be useful:

CLT for triangular arrays with homokedastic errors: Let $\{v_{i,n}, 1 \leq i \leq n, n \geq 1\}$ be a triangular array of identically distributed random variables. Assume that the random variables $\{v_{i,n}, 1 \leq i \leq n\}$ are jointly independently distributed for each n with $\mathbb{E}(v_{i,n}) = 0$ and $\mathbb{E}(v_{i,n}^2) = \sigma^2 < \infty$. Let $\{a_{ij,n}, 1 \leq i \leq n, n \geq 1\}, j = 1, ..., k$ be triangular arrays of real numbers that are bounded in absolute value. Further let

$$\mathbf{v}_n = \begin{pmatrix} v_{1,n} \\ \vdots \\ v_{n,n} \end{pmatrix}, \quad \mathbf{A}_n = \begin{pmatrix} a_{11,n} & \dots & a_{1k,n} \\ \vdots & & \vdots \\ a_{n1,n} & \dots & a_{nk,n} \end{pmatrix}$$

Assume that $\lim_{n\to\infty} n^{-1}\mathbf{A}_n^\top \mathbf{A}_n = \mathbf{Q}_{AA}$ is finite and nonsingular matrix. Then

$$\frac{1}{\sqrt{n}}\mathbf{A}_n^\top \mathbf{v}_n \xrightarrow{d} \mathrm{N}(\mathbf{0}, \sigma^2 \mathbf{Q}_{AA})$$