Universidad Católica del Norte, Economic Department

Microeconomics: Exam

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Some comments:

- If you believe a question is unclear, please state how you interpret the question.
- Please, use formal mathematical language wherever possible.
- You must show all work for partial credit to be awarded.
- Total points: 58.

1 Consumer Theory

1. (5) Assume that the consumer faces $\boldsymbol{x} = (x_1, x_2, x_3)$, such that $\boldsymbol{p} \gg 0$. We know that the Walrasian demand for good 1 and 2 are:

$$
\begin{aligned}
x_1(\boldsymbol{p}, y) &= \alpha_0 + \alpha_1 \frac{p_1}{p_3} + \alpha_2 \frac{p_2}{p_3} + \alpha_3 \frac{y}{p_3} \\
x_2(\boldsymbol{p}, y) &= \beta_0 + \beta_1 \frac{p_1}{p_3} + \beta_2 \frac{p_2}{p_3} + \beta_3 \frac{y}{p_3}
\end{aligned}
$$

We observe that for (p_1^0, p_2^0) the demanded quantities for x_1 and x_2 are $x_1^* = 1, x_2^* = 2$, whereas for other vector of prices (p_1^1, p_2^1) we observe that $x_1^{**} = 1, x_2^{**} = 1$. Compute the income-elasticity η_i for good 1.

2. (7) Let $\boldsymbol{x}(\boldsymbol{p}, y)$ be the consumer's walras demand system. Show that $\sum_{i=1}^{n} s_i \epsilon_{ij} = -s_j$, $j = 1, ..., n$, where ϵ_{ij} is the cross price elasticity and $s_i = p_i x_i(\boldsymbol{p}, y)/y$. (Hint: Use Walras's law and then differentiate both sides with respect to p_j.)

2 Firm Theory

1. Consider a firm that uses input 1, denoted x_1, to produce output denoted y, that is sold at a market price of p. Input 1 is purchased at a cost of w, In addition, the firm's production creates pollution that we have denoted as 2 (in amounts x_2) with price of 0. The firm use the following single output production function. Assume Free Disposal holds:

$$f(x_1, x_2) = \begin{cases} 0 & \text{if } x_1 < 1 \\ \sqrt{\min\{x_1 - 1, x_2\}} & \text{if } x_1 \geq 1 \end{cases}$$

   a) (2) Write the Lagrangian for the profit maximization problem.

   b) (5) Solve for the profit function for the firm given output price p and price w of input 1. Find π^* and comment the results.

   c) (7) Assume the firm is constrained to produce less than κ units of pollution (input 2). Solve for the firm's maximum profits as a function of p, w, and κ. (Hint: Try to use your intuition whether the constraint binds or not)

3 Partial Equilibrium

1. (5) A monopolist firm has the following production function $y = f(K, L)$. The market demand is $p = p(y)$ and assume that capital K is fixed in the short-run. Show that at the optimum we must have that: marginal revenue $\times$ marginal product of labor = marginal cost of labor.

4 Pure General Equilibrium

1. (7) Consider the following indirect utility functions for consumers A and B:

$$\upsilon^A(\boldsymbol{p}, y) = \ln y - \frac{1}{2}\ln p_1 - \frac{1}{2}\ln p_2$$
$$\upsilon^B(\boldsymbol{p}, y) = (p_1^{-1} + p_2^{-1})y$$

The initial endowments are $\omega_1^A = \omega_1^B = 5.8$ and $\omega_2^A = \omega_2^B = 2.1$. Compute the equilibrium relative prices in this economy. Assume that $p_1 = 1$. (Hint: Use Walras' Law and equlibrium's conditions!)

5 Applied Section

1. Assume you are interested in investigating the returns to scale in the mining sector. You propose to model the technology for firm i as $y_i = A_i x_{i1}^{\alpha_1} x_{i2}^{\alpha_2} x_{i3}^{\alpha_3}$, where x_1 is labor, x_2 is capital, x_3 is fuel and A captures unobservable differences in production efficiency. The cost function is:

$$c = r \cdot (A\alpha_1^{\alpha_1}\alpha_2^{\alpha_2}\alpha_3^{\alpha_3})^{-1/r}\, y^{1/r} w_1^{\alpha_1/r} w_2^{\alpha_2/r} w_3^{\alpha_3/r}$$

where $r = \alpha_1 + \alpha_2 + \alpha_3$.

a) (5) Derive an estimable form for the cost function.

b) (5) According to your model in part (a), state the null hypothesis of HD1 in the cost function in terms of the parameter of your model.

c) (5) How will you estimate your function in part (a) using **restricted** OLS?

d) (5) State the null hypothesis of constant returns to scale in terms of the parameter of your model.